Competition for the Development of a New Product:
Theoretical and Experimental Investigation

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Abstract

The competition between two firms involved in developing a new product is modeled as a two-person nonzerosum game in strategic form in which each firm invests part of its R&D budget in the development of the product. The firm investing the highest amount wins the competition and receives a commonly known, exogenously determined reward. Investments of both firms are assumed to be project specific and nonrecoverable, implying an unconditional commitment of resources. The case of symmetric firms with the same R&D budget and that of asymmetric firms with different R&D budgets are examined both theoretically and experimentally. The equilibrium solutions for both cases call for mixed strategies resulting in expected payoffs that are independent of the size of the reward, equal to the minmax payoffs, and Pareto-deficient. The experimental results reject minmax play, show weak support for the equilibrium solutions on the aggregate but not individual level, and provide evidence for iterative deletion of strongly dominated strategies.
1. Introduction

Given the substantial advantages accruing to a firm which first successfully develops a new product (e.g., Besanko, Dranove, & Shanley, 1996; Carpenter & Nakamoto, 1989), it is not enough for a firm to evaluate its investment in R&D strictly in terms of the expenses necessary to develop and market the product. Rather, in a competition between two firms for developing a new product, a strategically sophisticated firm needs to anticipate the likely investments of its competitor when determining its level of investment.

The competition among firms developing innovative products has traditionally been characterized as patent races. An important feature of these patent races is the unconditional commitment of resources to winning the competition. Economists have modeled this feature using first or second-price all-pay auctions (e.g., Amann & Leininger, 1996; Dasgupta & Stiglitz, 1980; Fudenberg, Gilbert, & Tirole, 1983; Leininger, 1991) in which the first firm to successfully develop the new product "wins" the patent race and obtains exclusive rights to market the product, whereas the losing firms get nothing. Resources invested in the competition are assumed to be project specific and, therefore, unrecoverable. "While this is a rather extreme characterization, it does highlight the often critical advantage that goes to the first innovator" (Besanko et al. 1996, p.589). More importantly, it emphasizes the need for firms to be strategically sophisticated. Firms engaged in a patent race must anticipate the R&D investments of their competitors and make investments that will preempt competition.

Our purpose is to begin a systematic experimental investigation of decision behavior in patent races using the all-pay auction model. We start this investigation by presenting a simple all-pay auction model of patent races with complete information, that incorporates some of the strategic considerations underlying R&D competitions. Then in a series of experiments we study the extent to which naive subjects behave in a strategically sophisticated fashion and examine how closely their behavior conforms to the model.
predictions.

We first consider the case of symmetric firms with equal investment budgets \(e_i = e_j\). formulate it as a noncooperative two-person game in strategic form, and then derive the symmetric Nash equilibrium solution for the game. In equilibrium, each firm \(k\ (k = i, j)\), should randomly choose the size of its investment according to a probability distribution (mixed-strategy) defined over the set of its investments \(\{0, 1, 2, \ldots, e_k\}\). Next we consider the case of asymmetric firms \((e_i \neq e_j)\). We show that in equilibrium each firm should only consider about half of its possible investments, after iteratively deleting the strongly dominated strategies, and then randomly choose among them according to two different probability distributions, one for the "strong" player (i) and the other for the "weak" player (j). Later we test the implications of the equilibrium solutions in a series of laboratory experiments.

The rest of the paper is organized as follows. Section 2 defines the problem. Section 3 analyzes the case of competition between symmetric firms, and Section 4 analyzes the case of competition between asymmetric firms. Section 5 details the experimental study conducted to investigate competition between symmetric firms, and Section 6 describes the experimental investigation of competition between asymmetric firms. Section 7 summarizes our findings.

**Problem Statement**

We consider the case of two firms competing in the development of the same product. The first firm to succeed obtains a patent and monopoly rights to the product, whereas the other firm gets nothing. Like O'Neil (1986), we assume a discrete model and commonly known budgetary constraints. We further assume that the amount spent on R&D determines the time required to develop the product, render it commercially viable, patent it, and bring it to market (e.g., Besanko et. al, 1996).

Formally, we consider two players (firms). i and j, with R&D budgets \(e_i\) and \(e_j\) respectively, which are assumed to be common knowledge. Each player \(k\ (k = i, j)\) invests the
amount $c_k$ in R&D ($c_k=0, 1, \ldots, e_k$). Investments are assumed to be made simultaneously.

Let $r$ denote the payoff associated with winning the competition (obtaining a monopoly and the patent rights); we assume that the value of $r$ does not depend on the actual investments $c_i$ and $c_j$, and that $r>e_k$. Let $s$ denote the amount that a player receives in the event that both firms make the same investment ($c_i=c_j$). The resulting payoff structure has the following form:

$$U_i(c_i, c_j) = \begin{cases} 
  r + e_i - c_i, & \text{if } c_i > c_j \\
  s + e_i - c_i, & \text{if } c_i = c_j \\
  e_i - c_i, & \text{if } c_i < c_j 
\end{cases}$$

and

$$U_j(c_i, c_j) = \begin{cases} 
  r + e_j - c_j, & \text{if } c_j > c_i \\
  s + e_j - c_j, & \text{if } c_j = c_i \\
  e_j - c_j, & \text{if } c_j < c_i 
\end{cases}$$

where $U_k(c_i, c_j)$ is the payoff of player $k$, given the investments $c_i$ and $c_j$, and $0 \leq s \leq r$.

The most common assumption in patent race models is that $s=r/2$; if the two firms invest (approximately) the same amount in the development of the new product ($c_i=c_j$), then the reward $r$ is shared equally between them. We depart from this model by assuming $s=0$. This assumption is tenable in situations in which the competing firms are in a strict duopoly, and simultaneously introducing the new product strictly cannibalizes existing product lines without increasing the overall market. It applies to patent races where the new product is introduced by both firms at the same time and, therefore, no patent is granted because of litigation. It is, perhaps, even more applicable to arm races in which the development of equal technologies at about the same time (e.g., ballistic missiles, nuclear power capability) results in a stalemate (and, consequently, total loss of the resources expanded in developing the technology). The second reason for the assumption that $s=0$ is that it drives the theoretical
results presented below, which imply randomization over all the possible values of $c_k$ when the players are symmetric ($e_i = e_j$) and iterative elimination of strictly dominated strategies when they are not. It is these non-obvious strategic implications that we wish to study experimentally in the context of a competition for the development of a new product.  

2. Symmetric Firms

The competition between the two firms is modeled as a noncooperative two-person game in strategic form with action spaces $C_i = \{0, 1, ..., e_i\}$ and $C_j = \{0, 1, ..., e_j\}$. Consider first the case of symmetric players, namely, $e_i = e_j$. Assuming risk-neutrality, Rapoport and Amaldoss (1996) show that this game has no pure strategy equilibrium solution but has unique mixed strategy equilibrium solution given by

$$p = (p_0, p_1, ..., p_e) = \left(\frac{1}{r}, \frac{1}{r}, \frac{1}{r}, ..., \frac{r - e}{r}\right). \tag{1}$$

Furthermore, in equilibrium each player's expected payoff is $e$.

There are three features of the equilibrium solution, all experimentally testable, that warrant discussion. First, note that the expected payoff of each player is independent of the reward $r$. In equilibrium, substantially increasing the reward for winning the competition affects the mixed strategy $p$, but not the expected payoff. Therefore, firms engaged in this race for product development should be indifferent to the magnitude of the reward associated with winning the competition, so long as it exceeds their R&D budget.

Second, note that by setting $c_k = 0$ each player can guarantee the value of his or her budget; $c_k = 0$ implies that $U_k = e_k$. The equilibrium strategies do not yield a higher payoff than $e$; indeed, they do not even guarantee the payoff $e$. As noted by Aumann and Maschler (1972), who observed the same phenomenon in two-person zero-sum games, if the equilibrium strategies are to be played at all, they are presumably played with the hope that each player will obtain his or her equilibrium payoff. But then, why play "with hope" when the two players could each guarantee the same payoff by choosing their minmax rather than
equilibrium strategies? Therefore, it is doubtful whether players will invest any positive amount in this new product development competition, and even if they do, whether they will play the mixed equilibrium strategy.

Finally, note that the equilibrium outcome is Pareto-deficient. Even without preplay communication, when the game is iterated in time, both players can increase their individual payoffs if both depart from equilibrium play. For example, suppose that the two players tacitly coordinate their actions and jointly alternate between 0 and 1. Then the expected payoff in this case is \((r+2e-1)/2\). Unlike the equilibrium expected payoff, this expected payoff increases linearly in \(r\). For the parameters used in Experiment 1, the difference between the two expected payoffs is substantial. (For example, if \(e=5\) and \(r=8\), then the expected payoff is 8.5 rather than 5; and if \(e=5\) and \(r=20\), then the expected payoff almost triples from 5 to 14.5). Expected payoffs exceeding the equilibrium payoffs can be obtained even without tacit coordination (e.g., by each player independently randomizing between \(c_x=0\) and \(c_x=1\) with equal probabilities). One may anticipate, then, that as the reward for winning the competition, \(r\), increases, players will deviate (e.g., by tacit agreement, if the stage game is iterated many times) from the equilibrium strategy and lower their investments in an attempt to maximize payoff.

Equation 1 states the equilibrium solution for risk-neutral players. Similar solutions can be derived for the general case, where various attitudes toward risk are allowed. (See Rapoport & Amaldoss, 1996.)

3. Asymmetric Players

The formulation of the investment game for asymmetric players is the same as in Section 2 with the only exception that \(e_i \neq e_j\). We assume without loss of generality that \(e_i > e_j\), and refer to firms \(i\) and \(j\) as the "strong" and "weak" players, respectively. As in the case of symmetric players, we assume that \(r > e_i > e_j > 0\).
When players are asymmetric, it is sufficient to consider the case \( e_i = e_j + 1 \). If \( e_i > e_j + 1 \), any strategy of player \( i \) dictating investment of \( c_i = e_i + h \) (\( h > 1 \)) is strongly dominated by the strategy \( c_i = e_j + 1 \). To simplify notation, we shall denote player \( i \)'s budget by \( e \) and player \( j \)'s budget by \( e-1 \).

When the players are asymmetric, their strategic considerations differ from the ones in the symmetric case. The strong player can guarantee himself a payoff of \( r \) by investing his entire budget, \( e \). Anticipating that, the weak player can invest zero (and receive a net payoff of \( e-1 \)). If, however, player \( i \) anticipates that player \( j \) will invest nothing, he is better off investing 1 (rather than \( e \)). But once the strong player does not invest his entire budget, he is no longer sure of winning.

Denote the strategy sets of players \( i \) and \( j \) by \( C_i = \{0, 1, \ldots, e_i \} \) and \( C_j = \{0, 1, \ldots, e_j \} \), respectively, and assume that players \( i \) and \( j \) are risk-neutral with respective budgets \( e \) and \( e-1 \). Then the equilibrium strategies of the two players are characterized as follows (Rapoport & Amaldoss, 1996):

If \( e \) is even, the strong player randomizes over his odd-numbered investments according to the probability vector

\[
p_i = (p_{1i}, p_{3i}, \ldots, p_{e+1i}) = (2/r, 2/r, \ldots, (r-e+2)/r),
\]

and the weak player randomizes over her even-numbered investments (including the investment \( 0 \)) according to the probability vector

\[
p_j = (p_{0j}, p_{2j}, \ldots, p_{e+2j}) = ((r-e+2)/r, 2/r, \ldots, 2/r).
\]

If \( e \) is odd, the strong player randomizes over his odd-numbered investments according to the probability vector

\[
p_i = (p_{1i}, p_{3i}, \ldots, p_{ei}) = (2/r, 2/r, \ldots, (r-e+1)/r),
\]

and the weak player randomizes over her even-numbered investments according to the probability vector

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\[ p_j = (p_0, p_2, \ldots, p_{e-1}) = ((r-e+1)/r, 2/r, \ldots, 2/r). \quad (5) \]

In equilibrium, the expected payoff of the strong player is \( r+1 \), if \( e \) is even, and \( r \), if \( e \) is odd. The expected payoff of the weak player, for both odd and even values of \( e \), is \( e-1 \).

The equilibrium solution for the asymmetric case shares most of the features of the solution for the symmetric case discussed above. However, when the players are asymmetric the expected payoff of the strong player depends on the reward, while the weak players' does not. Therefore, the strong player would prefer the reward to be as high as possible, whereas the weak player should be indifferent to the size of the reward.

As in the case of the symmetric game, the weak player can guarantee her budget, \( e-1 \), by not investing in the development of the product. Similarly, the strong player can guarantee the reward \( r \). When \( e \) is odd, as in our experiment in Section 5, the expected values of the two players under equilibrium play are \( e-1 \) and \( r \). However, the equilibrium solution, unlike minmax play, does not guarantee these values to the weak and strong players.

The most interesting implication of the equilibrium solution is that both players only play strategies that survive iterated deletion of strongly dominated strategies. Equations 2 through 5 imply that the number of iteratively undominated strategies for each player is about half of the number of his or her pure strategies. And the game theoretical analysis implies that the strongly dominated strategies of both players are iteratively eliminated in a specific order. If \( e \) is relatively large, it is highly unlikely that, without knowledge of noncooperative game theory and the possibility of inspecting and comparing all the entries in the payoff matrix, boundedly rational players can go through the elaborate cognitive process of iteratively eliminating all the dominated strategies. Stahl and Wilson (1995) and Nagel (1995), among others, report evidence that the number of iterations subjects perform when first presented with dominance solvable games is rather small; perhaps as few as one or two. If, indeed, there is a hierarchy of types of players who differ from one another in their depth of reasoning (Ho
& Weigelt, in press; Stahl, 1993; Stahl & Wilson, 1995), one may expect that a (possibly large) subset of the players will identify and properly eliminate the first dominated strategy in the sequence (0), a smaller subset will identify and eliminate the second element in the sequence (1), and so on. This hypothesis is tested in Section 5.

4. Experiment 1: Symmetric Players

Subjects: Thirty-six undergraduate and graduate students from the University of Arizona participated in the study. Subjects were recruited through advertisements placed on bulletin boards on campus and class announcements. They were promised monetary reward contingent on performance for participation in a decision making experiment. Subjects were run in groups; 18 subjects participated in Group 1 and 18 in Group 2.

Procedure: Each experiment consisted of a single session lasting about 2 hours. Upon arrival at the laboratory, the members of each group were randomly seated in 18 separate booths. Subjects read the instructions for the experiment on their individual computer terminals. Any form of communication during the experiment was not possible.

Subjects were randomly matched into pairs on each trial. In particular, on each trial the 18 group members were matched to form 9 pairs according to a predetermined assignment schedule. Subjects had no way of knowing the identity of their competitors on any given trial. The assignment schedule ensured that each subject would be matched with each of the other group members approximately the same number of times, and that a subject would not be paired with the same competitor twice in a row.

The instructions (see Rapoport & Amaldoss, 1996) explained and demonstrated the competitive investment game. At the beginning of each trial, the subjects were informed of the values of e, e, and r. These payoff parameters were fixed for the entire duration of the experiment. Once the subjects made their investments privately, the computer compared each player’s investment with that of the other member of her/his pair and determined the winner.
Ties were counted as losses. At the end of each trial, each pair member was informed of 1) the values of \( c_i \) and \( c_j \), 2) the winning player (i or j), and 3) his or her reward for the trial \( (U_k) \).

Each group participated in two experimental conditions (or games) in a within-subjects design. The payoff parameters for these two games were

**Low Reward Condition (Game L):** \( e_i = e_j = 5 \), \( r = 8 \).

**High Reward Condition (Game H):** \( e_i = e_j = 5 \), \( r = 20 \).

Each stage game was played repeatedly for 80 successive trials. The subjects in Group 1 were presented first (in Phase 1) with Game L (trials 1 -- 80) and then (in Phase 2) with Game H (trials 81 -- 160), for a total of 160 trials. The order of presentation of the two games in Group 2 was reversed.

Investments on each trial were made in terms of a fictitious currency called "franc". At the end of the experiment, the individual payoffs were totaled and converted into US dollars at the exchange rate of 80 francs = $1.00. In addition, subjects were paid a $5.00 show-up fee.

**Results**

**Payoffs:** In equilibrium, the expected payoff per trial is \( e=5 \). In actuality, the mean payoff per trial in Game L computed across subjects and trials was 4.91 francs (with a standard deviations of 0.60). The difference between predicted and observed mean payoff is not significantly different from zero (\( t = -0.15 \)). The mean payoff per trial in Game H was 6.84 francs with a standard deviation of 1.17. Although the mean payoff per trial seems higher than predicted, the difference between predicted and observed mean payoff is again not statistically significant (\( t = 1.58 \)).

**Strategy Profiles:** With each subject making 160 decisions in both games L and H, we have a total of 5760 choices. The analyses reported below were performed on these data.
Table 1 shows the frequencies (and proportions) of choice of the six pure strategies. Computed across subjects and trials, the frequencies are presented separately in terms of group (1 and 2) and game (L and H). The most prominent finding is that subjects in both games L and H did not choose to play their minmax strategies of zero investment (thereby eliminating uncertainty and guaranteeing a payoff of 5 francs). Rather, most of the subjects played all six strategies, though not exactly in accordance with the mixed-strategy equilibrium solution. As predicted, the strategy c=5 was chosen considerably more often than any other strategy. Also as predicted, the subjects chose to invest their entire budget significantly more often when the reward for winning the competition was increased from 8 to 20. But in contrast to the equilibrium prediction, the other strategies c=0, 1, 2, 3, and 4 were not chosen with equal frequencies. Rather, strategy c=0 was chosen more often than any of the other four strategies in both games L and H, whereas the latter four strategies were chosen approximately the same number of times. Table 1 shows that the two groups differed from each other in the frequencies of the six investment strategies, suggesting that order of presentation of the two games mattered.

---Insert Table 1 about here---

To test the effects of group, game, and their interaction, as well as to assess changes in choice frequencies due to learning, we focused on the number of times each subject invested his or her entire budget. Table 1 shows that, across groups and games (L and H), the entire budget was invested in more than 54 percent of all the trials. We divided the 80 trials in each game into 8 equal size blocks of 10 trials each, and then counted for each subject separately the number of times that he or she contributed the entire budget in each block (16 data points for each subject across the two conditions). The resulting frequencies were then subjected to a 2 x 2 x 8 group by game by block ANOVA with repeated measures on the second and third factors.
Both the group main effect ($F_{1,30} = 47.8, p<0.0001$) and game main effect ($F_{1,30} = 161.8, p<0.0001$) were highly significant. There was no main effect due to block ($F_{7,30} < 1$). Also, the two-way interaction effects involving the block factor were not significant either ($F_{7,30} < 1$ for the block by group interaction, and $F_{7,30} = 1.4, p<0.2$, for the block by game interaction). However, both the group by game two-way interaction effect ($F_{1,30} = 27.5, p<0.0001$) and the three-way group by game by block interaction effect ($F_{7,30} = 3.8, p<0.0004$) were highly significant.

Table 2 presents the mean proportion of trials that the entire budget was invested by group, game, and block. Inspection of the table shows that across both conditions, the subjects in Group 1 invested their entire budget more often than the subjects in Group 2. However, this effect only obtained when Game L preceded Game H. When the order of presentation of the two games was reversed, the opposite effect obtained. This group by game interaction indicates a tendency to lower the frequency of investing the entire budget from the first to the second phase of the experiment, regardless of the value of $r$, showing weak evidence for tacit collusion. Thus, whereas we observe no learning across groups within game, there is learning across the two phases of the experiment resulting in lower frequency of investment of the entire budget (and, consequently, higher payoffs) The significant three-way interaction effect further indicates learning within game but in a different direction for each group. Table 2 shows that in Game L the mean proportion of subjects investing the entire budget increased across blocks in Group 1 and decreased in Group 2. Similar learning trends can be discerned in the results of Game H.

**Individual Differences:** The results reported in Tables 1 and 2 pertain to aggregate, not individual data. In fact, they may conceal considerable differences in investment strategy profiles among subjects. For example, the group results in Table 1 are compatible with the existence of two distinct groups of subjects, those adhering to the mixed strategy and those
playing the minmax strategy of zero investment. Figure 1 displays the frequency distributions of the number of times the budget was invested by each subject in the 80 iterations of the stage game. The distributions are computed across the two groups (n=36) for each game separately. Figure 1 shows marked individual differences in both games L and H, with frequencies of investing the entire capital ranging all the way from 0 to 80. The equilibrium solution (Eq. 1), which pertains to individual rather than aggregate results (and assumes risk-neutrality), does not account for the strategy profiles of all the subjects. Nor is there any evidence for minmax play, which implies a relatively large proportion of subjects falling in the frequency class 0 -- 10.

--Insert Figure 1 about here--

Discussion

Several qualitative implications of the equilibrium solution are supported at the aggregate level. Symmetric subjects engaged in competition for developing new product, rather than staying away from the competition and taking the guaranteed payoff $e_x$. Subjects mixed their choices over all six strategies as predicted, chose the strategy of investing all the capital more often than any other strategy, increased the frequency of choice of this strategy as the reward for winning the competition increased, and earned on the average their equilibrium payoff. However, the proportions of trials on which the entire budget was invested in general diverged rather than converged to the equilibrium with experience. Moreover, strong sequential dependencies were detected (see Rapoport & Amaldoss, 1996, for details). Logit analyses (not reported here) indicate that in deciding whether to change the size of the investment on the previous trial, subjects were mostly affected by the opponent's choice of strategy on the previous trial and the outcome of the previous trial (Rapoport & Amaldoss, 1996).

All of these findings suggest that it is more sensible to model the behavior of our
subjects with a dynamic rather than a static model like the equilibrium solution. Dynamic models should account, at least qualitatively, for the trial-to-trial changes in strategy, the significant group and game effects, the considerable individual differences in strategy profiles, and the learning trends described above. In particular, there are strong indications for learning across phases that result in trends away from equilibrium. These suggest that the behavioral regularities reported above could possibly be accounted for by adaptive learning models assuming that changes in strategy are attributed to certain reinforcement mechanisms (e.g., Roth & Erev, 1995) or by models making use of the concept of best reply. We defer the development and testing of learning models to a subsequent paper, and turn next to the investigation of investment competition between two asymmetric firms.

5. Experiment 2: Asymmetric Players

**Subjects:** Thirty-six undergraduate and graduate students from the University of Arizona participated in the study. None of the subjects had taken part in Experiment 1. Subjects were recruited in the same way as in Experiment 1 through advertisements and class announcements. They were all promised monetary rewards contingent on performance. Subjects were run in two groups of 18 players each.

**Procedure:** The procedure was identical to that of Experiment 1 except for the following differences. Rather than having two games (L and H), only a single game was played with the parameter values $e_i=5$, $e_j=4$, and $r=10$. Each subject participated in 160 iterations of this stage game, 80 in the role of the strong player (player i) and 80 in the role of the weak player (player j). The subjects were divided into nine pairs with the pairing changing randomly from trial to trial. In Phase 1 (trials 1 -- 80) each subject maintained one role (strong or weak) and in Phase 2 (trials 81 -- 160) the opposite role was played. Consequently, in each phase, each subject was randomly paired with one of the nine members who were assigned the opposite role. Subjects did not know the identity of their opponent on
any given trial. Because of possible carry over effects of role (from weak to strong and vice-versa) and possible group differences (as observed in Experiment 1), we study separately four sets of nine players:

Set 1WS -- Group 1; order of play: (Weak, Strong)
Set 1SW -- Group 1; order of play: (Strong, Weak)
Set 2WS -- Group 2; order of play: (Weak, Strong)
Set 2SW -- Group 2; order of play: (Strong, Weak)

Results

Payoffs: In equilibrium, weak players are expected to earn on the average 4 francs per trial and strong players 10 francs. The mean payoff per trial for the weak and strong players was 3.66 and 9.49, respectively, with corresponding standard deviations of 0.50 and 0.59. The results for both the weak and strong players do not differ significantly from the predicted equilibrium payoff ($t < 1$ in both cases).

Strategy Profiles: All the analyses reported below were conducted on 5760 strategy choices (36 subjects by 160 decisions). Table 3 shows the frequencies (and relative proportions) of choice of each of the pure strategies $c_i$ and $c_f$. The results are presented separately by role. In each case the frequencies are computed across groups and order of play. Also presented in Table 3 are the equilibrium probabilities of choice for risk-neutral players (Eqs. 4 and 5).

--Insert Table 3 about here--

As in Experiment 1, the weak players in Experiment 2 did not always play their minmax strategy; rather, strategy $c=0_i$ was played only about 55.0 percent of the time. The strong players in Experiment 2 also did not play their minmax strategy all the time; rather strategy $c=5_i$ was played only about 54.7 percent of the time. In equilibrium, strategies $0_i$ and $5_i$ each should be chosen 60 percent of the time by risk-neutral players. Table 3 shows that,
indeed, strategy 0, was played as often as strategy 5, but both were chosen slightly less frequently than predicted. We cannot account for this small discrepancy between observed and predicted results in terms of risk-aversion (Rapoport & Amaldoss, 1996). The frequencies of choice of the other pure strategies by players in both roles provide mixed support to the equilibrium solution. As predicted, the iteratively dominated strategies 0, and 1, were chosen very infrequently (0.7 and 3.0 percent of the time, respectively). However, the other strictly dominated strategies 2, 3, and 4, were chosen more frequently (5.4, 13.6, and 12.5 percent, respectively). Table 3 seems to suggest that strongly dominated strategies which are deleted later in the sequence were chosen more often than those that should have been deleted earlier. We shall test this hypothesis with individual data below.

Similarly to Experiment 1, we divided the 80 trials in each phase of the game into 8 blocks of 10 trials each, and then counted for each weak player the number of times she invested zero capital in each block, f, and for each strong player the number of times he invested his entire capital in each block, f, To test the effects of group, order of play, and block, the frequencies of choice f, and f, were subjected to a 2 x 2 x 8 group by order of play by block ANOVA with repeated measures on the group and block factors. Two separate ANOVAs were conducted, one for the weak players and the other for the strong players. Table 4 presents the mean proportion of trials in which zero capital was invested by weak players and the entire capital by strong players. The table presents this information by group, order of play, and block.

---Insert Table 4 about here---

We report first the ANOVA results for the weak players. The order of play main effect (F,31 = 33.5, p<0.0001), order of play by group interaction effect (F,31 = 7.5, p<0.006), order of play by block interaction effect (F,31 = 3.0, p<0.005), and the triple interaction effect (F,31 = 3.1, p<0.003) were all highly significant. Neither of the other main or interaction effects,
none involving the order of play factor, were significant. Clearly, the presence or absence of experience in playing the other role for 80 trials was the main factor causing subjects in the weak role to change the proportion of zero capital investment. Table 4 shows that subjects experiencing the role of the strong player in Phase 1 increased the frequency of zero capital investment in Phase 2, when assigned the role of a weak player. Whereas the mean proportion of zero capital investment in Phase 1 was 0.49, the same mean proportion increased to 0.61 in Phase 2, converging to the equilibrium prediction. Table 4 also shows that the order of play effect was stronger in Group 1 than in Group 2. The order of play effect was relatively steady across blocks in Group 1, but in Group 2 the effect was reversed after about 40 trials. Although we have found no significant main effect due to group in Experiment 2, this result is qualified by the significant two-way and three-way interaction effects involving group. As we noticed in Experiment 1, we find additional evidence that groups develop their own dynamics of play, and that switching opponents from trial to trial may not be sufficient to remove the "population" effect.

Moving next to the ANOVA results for the strong players, we find again that previous experience with the other role, or lack of it, was the major factor causing the subjects to change the proportion of trials in which the entire capital was invested. Only the main effect due to order of play \((F_{1,31} = 17.4, p<0.007)\) and the order of play by group two-way interaction effect \((F_{1,31} = 122.5, p<0.0001)\) were significant. Table 4 shows that the subjects experiencing the role of the weak player in Phase 1 decreased the frequency of investing the entire capital in Phase 2, when assigned the role of the strong player. However, this effect was only manifested in Group 2.

**Individual Differences:** Figure 2 displays the frequency distributions, one for each role, of the number of trials in which zero capital (weak players) or the entire capital (strong players) was invested by individual subjects in the 80 iterations of the stage game. The
distributions are computed across both groups and both orders of play. Figure 2 displays widely dispersed distributions with a common mode at the frequency class 51 -- 60. A comparison of the two distributions by the Kolmogorov-Smirnov test for two samples shows no significant difference between them (p<0.01). Recall that the equilibrium prediction of investing no capital or the entire capital in 80 trials is 0.6 x 80 = 48. Using the normal approximation to the binomial distribution (with a significance level set at 0.01), we tested the equilibrium prediction with the individual data. The hypothesis could not be rejected for 14 of the 36 weak players and 18 of the 36 strong players.

--Insert Fig. 2 about here--

Iterative Elimination of Strictly Dominated Strategies: Equations 2 to 5 imply that only iteratively undominated strategies will be chosen. What informational assumptions are implicit in the statement of the proposition, and to what extent can we believe that these assumptions will be actually met by our subjects? Clearly, a player does not need to know anything about his or her opponent to decide that it can never be optimal to play a strongly dominated strategy. However, to justify deleting strategy $1_j$, player $j$ has to know that player $i$ will not choose the strongly dominated strategy $0_i$. Similarly, to justify deleting strategy $2_i$, player $i$ must assume that player $j$ will delete strategy $1_j$. In other words, player $i$ has to know that player $j$ knows that player $i$ will not play his strongly dominated strategy. More generally, to justify an arbitrary number of deletions of strongly dominated strategies, it is necessary to assume that it is common knowledge that no player is sufficiently irrational as to play a strongly dominated strategy (Brandenburger, 1992).

It has long been recognized that the assumption of common knowledge of rationality has strong implications. When the number of steps in the iterative deletion of strongly dominated strategies is relatively large, the common knowledge assumption would seem to be behaviorally unacceptable, as it imposes cognitive demands that may not be satisfied by
boundedly rational players. When the number of pure strategies is rather small, as in our two experiments, the full force of common knowledge is not needed. Therefore, we state and then test a weaker hypothesis concerning lower levels of knowledge of rationality. To state this hypothesis, we say that event $E$ has knowledge level of degree 1, if player $i$ knows it, of degree 2, if player $j$ knows that player $i$ knows it, of degree 3, if player $i$ knows that player $j$ knows that player $i$ knows it, and so on. The "level of knowledge of rationality" hypothesis simply states that the proportion of players with knowledge level of degree $h$ decreases in $h$. When applied to our game, this hypothesis implies that a fraction of the subjects will iteratively delete $h$ strongly dominated strategies, a smaller fraction will delete $h+1$ strongly dominated strategies, and so on for $h=1, 2, \ldots, 5$.

Indirect support for this hypothesis is already presented in Table 3, which shows that strategies $0, 1, 2, 3, 4$, which should have been iteratively deleted in this order, were chosen 19 (0.7%), 85 (3.0%), 156 (5.4%), 391 (13.6%), and 361 (12.5%) times, respectively. With a minor exception, these proportions increase in $h$ as hypothesized. Note that these are aggregate results which may mask considerable individual differences, whereas the hypothesis above refers to individual players.

To test the hypothesis about levels of knowledge of rationality on the individual level, we counted the number of subjects, out of 36, who chose any of the iteratively strongly dominated strategies on $m$ out of 80 trials. Table 5 presents the results. It shows (column 2) that 32 of the 36 subjects never chose strategy 0, when playing the role of the strong player for 80 successive trials, 2 subjects chose it only once, and 2 chose it at least five times (the exact frequencies are 6 and 11). Similarly (see column 3), 23 of the 36 weak subjects never chose strategy $1$, 3 chose it only once, and so on. Table 5 shows that the number of subjects who chose any iteratively strongly dominated strategy no more than 5 percent ($m=4$) increases in the order of deletion, thereby strongly supporting the level of knowledge hypothesis.
Subjects who approach the investment game strategically and iteratively delete strongly dominated strategies in their mind should do so whether they play the role of strong or weak player. To test the level of knowledge hypothesis across both roles, we counted for each subject the number of times, out of a total of 160 trials, that he or she played each of the following tuples of iteratively strongly dominated strategies: \((0, 0, 1, 1, 2, 2, 3, 3)\), \((0, 1, 1, 2, 2, 3, 3)\), and \((0, 1, 2, 2, 3, 3)\). The proportions of subjects who played these tuples no more than 8 times (5% of all trials) were 0.972, 0.889, 0.772, 0.500, and 0.194, respectively. Assuming a stricter criterion of 2.5 rather than 5 percent error \((m=4)\), the respective proportions decrease to 0.944, 0.806, 0.556, 0.306, and 0.083. These results provide strong support for the hypothesis about a hierarchy of levels of knowledge about rationality or depth of recursive reasoning in the population of our subjects.

Discussion

The equilibrium solution for the asymmetric investment game has two major implications. The first and weaker implication, which only invokes the notion of common knowledge of rationality, is that only iteratively undominated strategies will be chosen. Although this implication does not presuppose communality of beliefs or the same attitude to risk, it may still impose strong demands on the players' cognitive system when the number of iteratively dominated strategies is large. Evidence from sequential bargaining experiments (e.g., Roth, 1994) and the Centipede game (McKelvey & Palfrey, 1992) suggests that subjects typically do not use backward induction for iteratively deleting strongly dominated strategies in two-person games presented in extensive form, even if the number of stages in the game is limited and small. Actually, our data provide stronger support for a restricted process of iterative deletion of strongly dominated strategies, which postulates a hierarchy of classes of players who differ from one another in their level of knowledge of rationality.
The second implication is that both the strong and weak players will choose the strategies that are left after the iterative deletion according to two probability distributions, which are identical up to permutation. The results of Experiment 2 provide mixed support to this implication. The proportions of time that the strong players invested all of their capital and the weak players invested zero capital are practically the same. And the frequency distributions over players within a role who invested the entire capital (strong players) or zero capital (weak players) do not differ significantly from each other (Figs. 2A and 2B). Moreover, at the aggregate level, the actual proportions of zero or all capital investment come very close to the equilibrium solution. However, at the individual level, even if only these proportions are considered, the equilibrium solution fails to account for the investment choices of the majority of the subjects. Nor can it account for the sequential dependencies—the repetition bias—found on the aggregate and individual levels (Rapoport & Amaldoss, 1996). The sequential dependencies, outcomes of the logit analyses (see Rapoport & Amaldoss, 1996), and significant interactions involving the group factor all suggest some sort of an adaptive learning process in which players' strategy choices are influenced to different degrees by the payoffs of both players, the outcomes of the previous trial, as well as by beliefs about the opponent's future choice.

6. Conclusions

The race for new product development has been analyzed theoretically by industrial organization economists. With this theoretical framework in mind, we have attempted to experimentally investigate the competition between two firms for new product development using a very simple model with special strategic implications. Our results lend support to the hypothesis that firms would participate in new product development competition even in situations where staying away from the competition is equally profitable. Firms would seem to invest more aggressively as the size of the reward increases, and consequently dissipate
potential incremental profits. The latter result is in keeping with the spirit of Posner's (1975) rent dissipation hypothesis.

We also find that players iteratively delete strongly dominated strategies, but that the ability to do so varies considerably among players. In the asymmetric game, the weak player does not completely withdraw from competition, nor does the strong player completely dominate the weak player by persistent overinvesting. Rather, both agents play strategically, and their choice of strategies broadly conforms at the aggregate level to the equilibrium solution. These results add support to models that assume a restricted process of iterative deletion of strongly dominated strategies and the subsequent use of mixed strategies in the reduced game (e.g., Raju, Srinivasan, & Lal, 1990).

Finally, our results also show that, while at the aggregate level choices of strategies conform to the mixed strategy equilibrium solution, at the individual level subjects do not randomize over their strategies; rather, their choices depend to varying degrees on the history of the game. The appropriate adaptive learning model which may account for the major behavioral regularities reported in Experiment 2 is not obvious. With a judicious selection of their parameter values, reinforcement-based models (e.g., Roth & Erev, 1995) may account for the sequential dependencies, individual differences, and the choice of strongly dominated strategies. However, at present they cannot account for the iterative deletion of strongly dominated strategies, and in particular for the hierarchy of types of players. On the other hand, best reply adaptive learning models will not account for the choice of strongly dominated strategies or the considerable individual differences in our study. Adaptive learning models that combine ideas from these two classes of models might provide an answer.
References


Footnotes

1. We make no claim that the paradigm investigated in the present paper is an accurate model of R&D competitions for the development of new products. Rather, our purpose is to study this type of problem experimentally by first starting with a simple paradigm that, we believe, captures some of the basic strategic aspects of R&D competitions between either symmetric or asymmetric firms.

2. An earlier and considerably more extensive version of this paper appears in Rapoport and Amaldoss (1996), which is available upon request.

3. It should be emphasized that a randomized-strategy Nash equilibrium does not rely on any player flipping coins or otherwise choosing a strategy at random. Rather, player j’s randomized strategy is interpreted as a statement of player i’s uncertainty about j’s choice of a pure strategy (see, e.g., Binmore, 1992; Gibbons, 1992; Osborne & Rubinstein, 1994).

4. Because of the large number of statistical tests, we only interpret effects significant at the 0.01 level.
Table 1

Experiment 1: Mean Investment Profiles by Group, Game, and Condition

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<tr>
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</thead>
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<td>Game H</td>
<td>Game L</td>
</tr>
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<td>195 (0.14)</td>
<td>320 (0.22)</td>
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<td>93 (0.06)</td>
<td>168 (0.12)</td>
<td>64 (0.04)</td>
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<td>114 (0.08)</td>
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<tr>
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<td>58 (0.04)</td>
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<td>55 (0.07)</td>
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<td>1440</td>
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* Group 1 played first Game L and then Game H, whereas Group 2 played these two games in the opposite order.
Table 2

Experiment 1: Mean Relative Frequency of Subjects Investing the Entire Capital by Group and Block

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<td>0.56</td>
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<td>0.34</td>
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<td>0.66</td>
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* There are 18 subjects in each group and 10 trials in each block.
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* The frequencies are counted across both groups and order of play.
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Mean Relative Frequency of Trials in Which the Entire Capital Was Invested by Strong Players: by Group, Order of Play, and Block

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<th>Block</th>
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<td>0.54</td>
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### Table 5

**Experiment 2: Number of Subjects Choosing Iteratively Strongly Dominated Strategies (m Times out of 80 Trials)**

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<tr>
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<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
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</tbody>
</table>
List of Figures

Fig. 1A. Experiment 1: Frequency distribution of the number of times the entire capital was invested (r = 8).

Fig. 1B. Experiment 1: Frequency distribution of the number of times the entire capital was invested (r = 20).

Fig. 2A. Experiment 2: Frequency distribution of the number of times zero capital (weak players) was invested.

Fig. 2B. Experiment 2: Frequency distribution of the number of time the entire capital (strong players) was invested.
Figure 1A: Experiment 1: Frequency distribution of the number of times the entire capital was invested by symmetric players (r=8)

Figure 1B: Experiment 1: Frequency distribution of the number of times the entire capital was invested by symmetric players (r=20)
**Figure 2 A** Experiment 2: Frequency of the number of times the zero capital was invested by weak players

**Figure 2 B** Experiment 2: Frequency of the number of times the entire capital was invested by strong players
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