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Trust Building in Credence Goods Markets

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Abstract

We study trust building in credence goods markets in a dynamic setting. An extreme lemon problem arises in the one-shot game and results in no trade. In the repeated game, an expert's honesty is monitored through consumers' rejection of his recommendations. The expert's profit in the optimal equilibrium weakly increases in the discount factor but cannot achieve the first best, which contrasts sharply with the optimal equilibrium in experience goods markets. Competition enhances efficiency by allowing consumers to search for second opinions after rejecting the first treatment recommendation, but the efficiency gain comes at the cost of less honesty.

Keywords: credence goods, trust, efficiency

JEL: D80, D82, D83

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1 Introduction

In markets for professional services such as health care, consulting, taxi rides, and various repair services, buyers often lack expertise to identify the appropriate treatments for their problems and hence rely on experts for provision of both diagnosis and treatment. Information asymmetry may prevail even after consumption. For instance, consider that you have a knee injury and your doctor recommends knee surgery. You know whether the pain is relieved after the surgery, but it is hard to tell whether it could have been solved simply by RICE (rest, ice, compression, and elevation) and physical therapy. Similarly, if your car’s check engine light is on and the car mechanic repairs it by replacing an expensive part, you can hardly verify whether the problem is caused by a loose gas cap and could have been solved by simply replacing the gas cap.

Credence-good sellers’ expertise in diagnosing buyers’ problems makes them “experts”, but also provides them with opportunities to exploit their consumers. There is a great deal of documented evidence to demonstrate that expert fraud is very common and costly. CNN reported that Dr. Farid Fata in Michigan made millions of dollars from prescribing cancer treatment drugs to more than 500 patients who did not need them.¹ In a field experiment, Schneider (2012) found widespread under- and over-treatment in the auto repair market. Blafoutas et al. (2013) and Liu et al. (2018) find empirical evidence that taxi drivers take nonlocals for unnecessary detours.

When consumers are concerned about expert fraud, trust plays an important role in facilitating trade and improving expert profits. If consumers trust that an expert will make honest recommendations, they are willing to pay more for his services. Despite the importance of trust in credence goods markets, the mechanism through which a credence-good seller builds trust in a repeated game has not been thoroughly studied in the literature and is not well understood.

In this paper, we adopt the notion of trust in the repeated-game literature (see e.g., Cabral, 2005 for definition of trust and how it differs from reputation). More specifically, in the context of credence goods markets, consumer trust is defined as consumers’ belief of the expert’s level of honesty in reporting the nature

¹According to “Patients give horror stories as cancer doctor gets 45 years in prison”, CNN, July 11, 2015, many non-cancer patients received expensive and aggressive treatments but only learned that they never needed those treatments after Dr. Fata’s arrest in 2013.

of their problems. The fact that trust building for credence-good sellers in a dynamic setting has not received the attention it deserves may be driven by the presumption that trust building mechanism for credence goods is similar to the well-researched trust building mechanism for experience goods.² Nevertheless, we argue that these trust building mechanisms are quite different. Experience-good buyers can monitor the quality of the good based on their consumption experiences. However, such monitoring technology is ineffective for credence goods because consumption and monitoring for honesty go against each other. Once buyers have received the services and their problems have been fixed, they cannot tell whether they really needed the service, i.e., whether the expert was honest.

We ask the following research questions: 1) How does a credence-good seller gain consumer trust in a repeated-game setting? 2) How do the seller's concerns for future business shape his conduct? 3) Is the efficient outcome attainable when the expert is sufficiently patient? 4) How does competition influence the trust building mechanism, market efficiency, and expert honesty?

In our model, a long-lived expert interacts with a sequence of short-lived consumers, each with a problem causing either a substantial or a minor loss. We thereafter call the problem *serious* or *minor*, respectively. Consumers do not know the nature of their problems and rely on the expert for diagnosis and treatment. The expert has one treatment which can fix both types of problem. It is efficient to use the treatment for the serious problem but inefficient to use it for the minor problem. This assumption allows us to analyze both over- and under-treatment.

In each period, the expert posts a price for his treatment.³ A consumer arrives in the market. She observes the price and decides whether to consult the expert for treatment. Through diagnosis, the expert learns the nature of the consumer's problem. Then, he either recommends the treatment at the quoted price

²For an excellent review of the voluminous literature on reputation and trust of experience goods sellers, please see Bar-Isaac and Tadelis (2008).

³We focus on single price for tractability. In general, the expert can post two prices, one for each type of problem. In our setting, allowing for two prices is unlikely to help the expert make more profit because the price meant for the minor problem will be rejected, given our assumption that it is inefficient to fix the minor problem. In many practical situations, professions do not provide further classification for a procedure. For example, because there is only one type of coronary angioplasty in the medical profession, a hospital charges one price for angioplasty and does not tailor the price of angioplasty based on the severity of the recipient's heart disease although the hospital bears a higher expected cost of providing the procedure to higher risk patients. (See the Havard business School case "The Medicines Company" by John T. Courville.) We focus on these situations.

or recommends no treatment. If the expert recommends the treatment, the consumer decides whether to accept it. Once the consumer accepts the treatment, the expert is liable for fixing the consumer’s problem. At the end of the period, the expert’s price, his recommendation, the consumer’s acceptance decision as well as her utility from accepting or rejecting the expert’s recommendation become public information. In other words, the consumer shares her experience with others. This simplifying assumption is adopted for tractability.⁴ It will be clear that even with this simplifying assumption, monitoring is still imperfect.

We assume the expected loss caused by the problem is lower than the cost of fixing the serious problem.⁵ Under this assumption, an extreme “lemon problem” emerges in the one-shot game. The expert cannot gain consumer trust when he has no concerns for future business and the market collapses. This no-trade equilibrium outcome holds whether the services are credence goods or experience goods because consumers’ post-consumption experiences do not play any role in the one-shot game.⁶

We characterize the set of perfect public equilibrium payoffs of the repeated game for any discount factor using the approach pioneered by Abreu, Pearce and Stacchetti (1990). We focus on the most profitable equilibrium and show that it is also the most efficient equilibrium. This equilibrium is referred to as the *optimal equilibrium* thereafter. Since the expert’s recommendation history is publicly observable, players’ strategies can depend on the entire recommendation history and hence the strategy space is huge. Nevertheless, the expert can implement the optimal equilibrium using simple stationary strategies. As the discount factor increases, the expert’s equilibrium profit gradually increases but cannot attain the first best. For any discount factor less than one, the optimal equilibrium either involves *overtreatment* for the minor problem or *undertreatment* for the serious problem. This stands in sharp contrast with canonical models of experience goods markets wherein the first best is attained when the discount factor is greater than a threshold.⁷

⁴This assumption can be justified by word of mouth communication among consumers.

⁵This assumption is not crucial to our main findings in the repeated-game setting. This issue will be discussed in Section 6.

⁶In our main model, expert service is modeled as a credence good. In Section 3, we formally specify an alternative model in which the expert service is an experience good. By then, the distinction between credence expert service and experience expert service will be clear.

⁷We formally demonstrate this result in a benchmark model of experience services markets in Section 4.1.

When the expert is sufficiently patient, in the optimal equilibrium he makes honest recommendations but charges a price high enough to induce consumers to sometimes reject the treatment. Consumer rejection is necessary for them to detect cheating and for the expert to prove his honesty. To see this, let's revisit the examples of knee injury and car repair. Suppose that you reject the knee surgery recommended by your doctor and instead try resting and exercise therapy at home. If your pain is relieved after some time, the injury must be minor and you can infer that the doctor has made an unnecessary treatment recommendation. However, if your pain gets worse and you start to experience limited range of motion, the injury must be serious and you learn that the doctor has been honest about your condition.⁸ Similarly, if you refuse to take the car mechanic's recommendation to replace the engine sensor and your car turns out to be fine, you can infer that the car mechanic exaggerated the problem. But if your engine stops to work after a while, the mechanic must have made an honest recommendation.

If the expert is proven to be honest, he gains the trust from future consumers who will continue to accept his recommendations with a positive probability. Otherwise, the expert loses consumers' trust and all future business. We refer to this equilibrium as the *monitoring-by-rejection equilibrium*. The expert wants to induce some rejection from consumers because honesty can only be monitored by rejection, which helps the expert gain trust of future consumers and continue to charge a high price. Although consumer rejection keeps the expert honest, it also results in inefficient undertreatment for the serious problem. Nevertheless, the efficiency of the equilibrium gradually increases in the discount factor and converges to (but never achieves) the first best because a smaller and smaller (yet still positive) rejection rate is required to keep the expert honest when he is more and more patient.

When the expert becomes less and less patient, it is too costly for him to induce monitoring for honesty because it requires a higher and higher rejection rate of recommended treatment for monitoring-by-rejection to be effective. In the optimal equilibrium, the expert posts a low price for his treatment and recommends the treatment to both types of problems. The expert takes a loss from efficiently treating the serious problem

⁸A study by Sihvonen R. et al. (2013) found that for many patients, arthroscopic surgery is no more effective at relieving symptoms than rest, exercise and over-the-counter pain killers. The study is reported in the Atlantic and CBS news. See "When Evidence Says No, but Doctors Say Yes," and "Common arthroscopic knee surgery no better than "sham" version, researchers say".

but makes a profit from inefficiently fixing the minor problem. Consumers always accept the expert's recommendations. We refer to this equilibrium as the *one-price-fix-all equilibrium*. This equilibrium is characterized by a low level of trust because consumers expect the expert to always recommend the treatment and never honestly report the nature of their problems. Although the expert has an incentive to cherry-pick the minor problem to treat, his refusal to treat the serious problem will trigger the punishment phase in which he loses future business. Since the expert gives up inducing monitoring for honesty, the minor problem is fixed with probability one in equilibrium, causing inefficiency from overtreatment. Consumers' failure to monitor expert honesty through consumption stands in sharp contrast with experience goods market in which consumption facilitates monitoring the quality of the goods.

Our main model shows that the promise of future business does not warrant efficient treatment in credence goods markets when the expert is monopoly. It is natural to ask how competition affects trust building, market efficiency and honesty. We extend the monopoly model to a competitive market consisting of n experts with free entry and exit. There exists a symmetric equilibrium in which consumers reject the first treatment recommendation with a positive probability and incur a search cost to solicit second opinions after the rejection. In other words, they use rejection of service and search for second opinions to monitor expert honesty. In this equilibrium, all experts lie with a small probability. When a consumer samples multiple opinions, she can identify cheating experts through observing conflicting recommendations. If an expert is caught cheating, he loses all future consumers, exits the market, and is replaced by a new expert. This competitive equilibrium is sustainable when the discount factor is high and the search cost is low.

The competitive equilibrium has some interesting features. First, competition can eliminate undertreatment but overtreatment still arises. A skeptical consumer may reject the first treatment recommendation, but she accepts the treatment on her second visit if the second opinion confirms the first opinion. Thus, the serious problem is repaired with probability one. Nevertheless, overtreatment occurs when both the first and the second opinions exaggerate the consumer's problem and recommend the treatment.

Second, when the search cost is sufficiently low, competition can improve market efficiency through a more efficient monitoring technology for honesty. In the monopoly market, monitoring for honesty comes

at the expense of undertreatment through consumer rejection. In the competitive market, monitoring for honesty still requires consumer rejection but undertreatment is avoided because of consumers' search for second opinions. The competitive equilibrium becomes more efficient when the search cost is reduced. In the limiting case in which the search cost converges to zero, the competitive equilibrium converges to the first best.

Third, the gain in efficiency comes at the expense of honesty. Given any positive search cost, the competitive equilibrium involves small but pervasive cheating even for discount factors arbitrarily close to one. By contrast, the monopoly expert makes honest recommendations when the discount factor is sufficiently high. This is because efficiency gain is realized only when consumers search for second opinions, but consumer search is not compatible with honest recommendation: If experts all make honest recommendations, consumers will not incur the search cost to sample another opinion.

Our analysis shows that although reputation mechanism facilitates trade in credence goods markets, it is less effective than in experience goods markets. While honesty and efficiency can be jointly achieved in experience goods markets when the discount factor is greater than a threshold, the first best is not attainable in credence goods markets for discount factors arbitrarily close to one. We also show that competition can enhance the reputation mechanism in credence goods markets through consumer search for second opinions. Hence, policies that make second opinions readily available and that encourage consumer search can improve efficiency. By contrast, ability to search for second opinion in experience goods markets has an opposite effect. We show that it limits the price experts can charge for the service and as a result renders reputation harder to sustain.

Most of the existing literature on credence goods focuses on a one-time transaction between experts and consumers and studies different mechanisms that discipline expert behavior. Contributions in this regard include Pitchik and Scotter (1987), Wolinsky (1993), Fong (2005), Liu (2011), Emons (1997, 2001), Alger and Salanie (2006), Dulleck and Kerschbamer (2009), Fong et al. (2014), Bester and Dahm (2017), Chen et al. (2018). Dulleck and Kerschbamer (2006) provides a comprehensive review of the early literature in one-shot games.

Despite the extensive studies of credence goods in one-shot games, the role of trust in a dynamic setting of credence goods has not been as thoroughly investigated. The paper most closely related to ours is Wolinsky (1993) who investigates experts' reputation concerns in Section 5 as an extension. There are important differences between his analysis and ours. First, Wolinsky (1993) rules out supergame consideration. He considers an overlapping generation model where consumers live for two periods and monitor experts by their own past experiences. We allow a consumer to observe the entire public history and base her strategy on it. Second, in his analysis of reputation concerns, Wolinsky assumes that consumers delegate treatment decisions to experts and commit to accepting any bill presented to them. We relax this assumption and allow consumers to reject a treatment recommendation and use rejection and/or second opinions as monitoring technologies.

Ely and Välimäki (2003), Frankel and Schwarz (2009), and Fong and Liu (2018) investigate a long-lived expert's recommendation strategy when facing a sequence of short-lived consumers. Similar to Wolinsky's (1993) analysis of experts' reputation concerns, Ely and Välimäki (2003) and Frankel and Schwarz (2009) assume that consumers commit to accepting the chosen expert's recommendations, so they also cannot use consumer rejection or search for second opinions to monitor the honesty of experts' recommendations. Fong and Liu (2018) study the effect of Liability on the reputation mechanism which we do not consider in this study. Moreover, Fong and Liu (2018) only consider the monopoly credence goods markets and do not compare the reputation mechanism in credence goods markets and in experience goods markets which is central to our paper.

Taylor (1995) considers a dynamic model in which experts have incentives to recommend unnecessary services to a durable good whereas the owner of the good has incentives to shirk in maintenance efforts. Taylor (1995) shows that short-term contract may give rise to low maintenance efforts or strategic delay in checkups. When short-term contract is inefficient, long-term contract in the form of actuarially fair insurance may implement the first best. Our paper focuses on experts' cheating incentives and does not consider consumers' moral hazard problem. Moreover, we analyze expert equilibrium behavior in both monopoly and competitive markets and highlight how competition affects honesty and efficiency while Taylor (1995) only considers the competitive market. Lastly, contrasting Taylor's (1995) setting where the good is either

diseased or healthy, in our setting, consumers are always diseased but sometimes it is inefficient to fix their problems. So, an insurance contract which ensures to fix consumers' problems does not result in the first best because there will be overtreatment for the minor problem.

Pesendorfer and Wolinsky (2003) study experts' incentives to make an unobservable and costly diagnosis effort for a consumer who searches for correct diagnosis in a competitive market. Experts in their model have no concerns for future business and they focus on experts' moral hazard in diagnosis, without considering the possibility that experts lie about their diagnosis. By contrast, we study a repeated game and focus on how experts' concerns for future business affect their incentives to honestly report their diagnosis.

In two interesting recent papers, Hafner and Taylor (2018) and Bardey et al. (2018) examine an expert's incentive to acquire information and truthfully report the information to their clients. In Hafner and Taylor (2018), clients need the expert's advice to decide how much to invest in their projects. In the dynamic setting, inefficiency in investment arises because clients ignore the externality of their investment on their successors. In Bardey et al. (2018), the client relies on the expert's recommendation to choose the product that best fits her needs. In a dynamic version of their setting, the threat of consumer switching can incentivize experts to acquire and reveal information. The expert's honesty is monitored very differently in our setting than in theirs. In Hafner and Taylor (2018), a client privately learns the accuracy of the expert's advice from the outcome of her project and makes a public referral which is used by successors to monitor the expert's advice. The client in Bardey et al. (2018) is long-lived and she learns the quality of the advice through purchase. In other words, the experts' services in both papers are experience goods instead of credence goods, and switching in Bardey et al. (2018) is used as a threat instead of a monitoring device. By contrast, in our model of credence services, clients monitor expert honesty through rejection or both rejection and search for second opinions.

There is an emerging body of literature studying the impact of reputation on expert conduct in laboratory experiments. Dulleck et al. (2011) found that reputation has no significant influence on expert honesty and Mimra et al. (2016) found that price competition undermines reputation building. In the above experiments, consumers delegate treatment decisions to their chosen experts and monitor the experts only through the

pattern of their past recommendations as in Wolinsky (1993), Ely and Välimäki (2003), and Frankel and Schwarz (2009). Based on a different setting, our findings suggest that consumer rejection and search for second opinions are important channels for monitoring expert honesty and facilitating trust building. It would be interesting to test in laboratory our findings that when consumers can choose whether to accept or reject a treatment recommendation, i) in a monopoly setting, when the expert is more patient, he is more likely to charge a high price and make honest recommendations, and ii) for high discount factors and low search costs, competition and consumers' search for second opinions improve market efficiency at the expense of honesty.

Our paper is broadly related to the literature of reputation building in markets for experience goods including Klein and Leffler (1981), Hörner (2002) and Park (2005). For an excellent review of the voluminous literature on reputation and trust of experience goods sellers, please see Bar-Isaac and Tadelis (2008). In these papers, consumers learn the value of the goods through consumption. Our paper differs from these papers in that consumption of the service does not generate any information about sellers' honesty.

To the best of our knowledge, we are the first to systematically compare monitoring technologies and the equilibrium outcomes of credence expert services and experience expert services in a dynamic setting. Our analysis reveals that an expert's concerns for future business impact the equilibrium outcome differently when the expert service is a credence good versus an experience good.

2 Model

Environment and Players A risk neutral, long-lived expert interacts with an infinite sequence of risk neutral, short-lived consumers. In each period $t \in \{1, 2, \dots, \infty\}$, one consumer arrives with a problem which is either minor or serious. Denote by l_s the loss from the serious problem and l_m the loss from the minor problem, with $0 < l_m < l_s$. We refer to l_s and l_m as substantial and minor losses, respectively. It is common knowledge that the problem is serious with probability $\alpha \in (0, 1)$. The expert can perfectly diagnose the consumer's problem at zero cost. There is one treatment available for the expert to fix both types of problems. It costs the expert c to apply the treatment on the serious problem and $c - \varepsilon$, for some $\varepsilon > 0$, to apply it

on the minor problem.⁹ We assume $l_m < c - \varepsilon < c < l_s$, so it is socially efficient to fix the serious problem but inefficient to fix the minor problem. This assumption allows us to study overtreatment, i.e., provision of a treatment whose cost outweighs its benefit to consumers, which is well documented in health care, car repair and legal services markets.

For algebraic simplicity, we assume $E(l) \equiv \alpha l_s + (1 - \alpha) l_m < c$, which implies $\alpha < \hat{\alpha} \equiv \frac{c - l_m}{l_s - l_m}$. Under this condition, there is no trade in the one-shot game. When this condition is violated, both types of problems will be fixed at the consumer's expected loss in the one-shot game. We discuss the case $\alpha \geq \hat{\alpha}$ in Section 6. Following the literature,¹⁰ we adopt the assumption the expert is liable for fixing the consumer's problem once the consumer has accepted his recommendation.

Payoffs The expert maximizes his expected discounted sum of profit with discount factor $\delta \in (0, 1)$. The expert's profit from treating the serious problem at price p is $p - c$, and that from treating the minor problem is $p - c + \varepsilon$. Each consumer maximizes her expected payoff. A consumer's payoff is $u = -l_i$ if she has problem $i \in \{m, s\}$ and the problem is left untreated. The consumer's payoff is $u = -p$ if the problem is fixed at price p . Note that under the liability assumption, the consumer receives the same utility once she accepts the treatment. The consumer's utility depends on the nature of her problem only when the problem is left untreated and the loss of it is realized.

Information The consumer does not know the nature of her problem and has to rely on the expert for diagnosis and treatment. The expert learns whether the consumer's problem is serious or minor through diagnosis. The cost of treatment incurred by the expert is unobservable to the consumer. At the end of a period, the prices charged by the expert, his recommendation, the consumer's acceptance decision as well as her utility become public information.

Timeline We summarize our model by describing the timeline of events in each period $t = 1, 2, \dots$

⁹In many real-life situations, the cost of a treatment depends on the complexity of a client's problem. For example, a cardiac surgeon spends less time on a by pass surgery when a patient's problem is mild than when her condition is serious. Similarly, it requires a tax lawyer less effort to file tax return for a taxpayer with a relatively simple tax situation.

¹⁰Pitchik and Schotter (1987), Wolinsky (1993), Fong (2005) and Liu (2011), Dulleck and Kerschbamber (2006).

Stage 1 The expert posts a price p_t for his treatment.

Stage 2 A consumer arrives in the market. Nature draws the loss of her problem according to the prior distribution of problems.

Stage 3 The consumer observes the price and consults the expert who perfectly diagnoses her problem and either proposes to fix the problem at the quoted price or refuses to treat the consumer.

Stage 4 If the expert offers to fix the problem, the consumer decides whether or not to accept the offer.

Stage 5 The expert and subsequent consumers observe the realization of a public randomization device, denoted by x_t .¹¹

Strategy and Equilibrium Concept Denote by $R_t \in \{p_t, \emptyset\}$ the recommendation made by the expert in Stage 3, where p_t denotes a recommendation of treatment and $\emptyset$ denotes refusal to treat the consumer. The expert's recommendation policy is β_{it} , $i = m, s$, where β_{it} denotes the probability that the expert recommends p_t for problem i and $1 - \beta_{it}$ is the probability that the expert recommends no treatment for problem i . Denote by $a_t \in \{0, 1\}$ the consumer's acceptance decision, where 0 denotes rejection and 1 denotes acceptance. Let $\gamma_t \in [0, 1]$ denote the probability that the consumer accepts price p_t . Formally, we denote $h_t = \{p_t, R_t, a_t, u_t, x_t\}$ as the public events that happen in period t and $h^t = \{h_n\}_{n=1}^{t-1}$ as a public history path at the beginning of the period, with $h^1 = \emptyset$. Let $H^t = \{h^t\}$ be the set of public history paths till time t . A public strategy for the expert is a sequence of functions $\{P_t, \beta_{mt}, \beta_{st}\}_{t=1}^\infty$, where $P_t : H^t \rightarrow \mathbb{R}_+$ and $(\beta_{mt}, \beta_{st}) : H^t \cup \mathbb{R}_+ \cup \{m, s\} \rightarrow [0, 1]^2$. The public strategy of the consumer is $\gamma_t : H^t \cup p_t \rightarrow [0, 1]$.

Equilibrium Concept We focus on *Perfect Public Equilibria* (Henceforth PPE) in which the expert and consumers use public strategies and the strategies constitute a Nash equilibrium following every public history. It is without loss of generality to restrict attention to public strategies. In our game the expert has private information about consumers' problems whereas consumers do not have any private information. In repeated-game terminology, it is a game with a *product monitoring structure*. Mailath and Samuelson (2006)

¹¹The use of public randomization device is common in the repeated game literature. This allows the expert and subsequent clients to publicly randomize at the beginning of the next period and hence will convexify the equilibrium payoff set.

have shown that every sequential equilibrium outcome is a perfect public equilibrium outcome in this case; so, there is no need to consider private strategies.

3 One-shot game

We first show that market collapses when the expert has no concerns for future business.

Lemma 1 *There is no trade in the static equilibrium.*

Given that it is efficient to fix the serious problem but inefficient to fix the minor problem, whenever trade happens, the serious problem must be repaired with a positive probability. For the expert to be willing to treat the serious problem, he must charge a price $p \geq c$. However, the consumer will reject such a price because once she accepts the price with a positive probability, the expert will recommend the treatment to both types of problems. This will yield the consumer a negative expected payoff given $E(l) < c \leq p$. The assumption $E(l) < c$ imposes an upper bound on the likelihood of the serious problem. Under this condition, an extreme “lemons problem” arises. The expert cannot gain trust from consumers and the market for the expert’s services completely shuts down, causing *undertreatment* of the serious problem.

We close this section by pointing out that the no-trade outcome continues to hold even if the expert’s service is an experience good. To make this statement precise and also for later reference, we define experience expert service as follows:

Experience Service by Expert *The expert’s service is either of high match value or of low match value to the consumer. The probability of high match value is $\alpha \in (0, 1)$, and the match value is the expert’s private information prior to the consumer’s consumption of the expert’s service. When no treatment is provided, both players’ payoffs are zero. When the match value is high, it costs the expert c to serve the consumer and his service provides a benefit of $l_s > 0$ to the consumer. When the match value is low, the cost to the expert and benefit to the consumer are respectively $c - \varepsilon$ and l_m . The consumer learns her match value after consumption because $l_s \neq l_m$.*

When the expert provides experience services, the consumer learns the match value of the expert's service through her post consumption payoffs. It is easy to see that under the same assumptions on the parameter values, there is still no trade in equilibrium, because *there are no more moves in the game after the consumer learns her actual problem*, so whether the consumer learns her actual problem or not has no bearing on the players' earlier actions. In fact, the observation that there is no difference between the static equilibrium in monopoly market for credence goods and that for experience goods holds in general. We show in Section 6 that this statement remains true when $\alpha \geq \hat{\alpha}$.

4 Repeated game

In this section, we study how the expert's conduct is affected by his concerns for future business. Because the expert is a monopolist who moves first in each period, we focus on the equilibrium which yields the expert the highest profit. We will show that the most profitable equilibrium is also the most efficient equilibrium. To illustrate how the monitoring technology for credence services differs from that for experience services and to explore its implication, we first discuss the optimal equilibrium in experience services markets. The characterization of the optimal equilibrium in credence services markets follows.

4.1 Benchmark: experience service markets

We begin with the experience service markets as defined in the previous section and demonstrate that honesty and efficiency are jointly achieved when the discount factor is above a threshold. To see this, consider the following strategies: In each period, the expert posts price l_s for the treatment and recommends the treatment only for the serious problem. Consumers accept the expert's recommendations as long as he did not recommend the treatment for the minor problem in the past. The game reverts to the static Nash equilibrium perpetually, otherwise. Given the expert's strategy, it is the consumer's best response to accept the treatment recommendation with probability one, which yields the expert an average profit of $\alpha(l_s - c)$. Since this is the surplus from the first best, it is the highest attainable profit for the expert.

Now, consider that the expert recommends the treatment to the minor problem. He receives a profit $l_s - c + \varepsilon$ in the current period but will lose all future business because his fraudulent recommendation will

be detected with probability one and hence he will be punished from the next period onward. The no lying condition requires

$$\underbrace{\frac{\delta}{1-\delta}\alpha(l_s - c)}_{\text{future loss}} \geq \underbrace{l_s - c + \varepsilon}_{\text{current gain}} \quad (1)$$

which is satisfied if

$$\delta \geq \underline{\delta}^e(\alpha) \equiv \frac{l_s - c + \varepsilon}{l_s - c + \varepsilon + \alpha(l_s - c)}. \quad (2)$$

So, for a given α , the expert can achieve the first best profit by making honest recommendations when the discount factor is greater than the cutoff $\underline{\delta}^e(\alpha)$. This is a common property of experience good models with perfect monitoring, with some difference in details. Here, the current gain from cheating is the one-period profit from treating the consumer's minor problem while charging her for the benefit of treating the serious problem, whereas in a standard experience good setting, the current gain from cheating is the cost saving from producing the low-quality instead of the high-quality good.

4.2 Optimal equilibrium in credence goods markets

Now, we characterize the optimal equilibrium in markets for credence services. We show that for a given parameter configuration, the optimal equilibrium is either the so called “monitoring-by-rejection equilibrium” or the “one-price-fix-all equilibrium”. We first characterize these two equilibria. Then, we prove that the expert's profit in any perfect public equilibrium is bounded above by his profit from one of these two equilibria. We also show that the most profitable equilibrium is also the most efficient one.

Monitoring-by-rejection equilibrium We begin by characterizing the monitoring-by-rejection equilibrium in the following proposition. Because the monitoring-by-rejection equilibrium is stationary, we suppress the subscript t .

Proposition 1 *In the monitoring-by-rejection equilibrium, the expert posts $p = l_s$ for his treatment; he recommends the treatment for the serious problem and no treatment for the minor problem, i.e., $(\beta_m, \beta_s) = (0, 1)$. The consumer accepts the treatment recommendation at $p = l_s$ with probability $\gamma^* = 1 - \frac{1-\delta}{\delta\alpha} \frac{l_s - c + \varepsilon}{l_s - c}$ as long as in the past the expert was not caught i) recommending the treatment for the minor problem, or ii) refusing to fix the serious problem, or iii) deviating in price. Otherwise, the game perpetually reverts*

to the static Nash equilibrium in which there is no trade. The average profit of the monitoring-by-rejection equilibrium is

$$\pi^m \equiv \alpha(l_s - c) - \frac{1 - \delta}{\delta}(l_s - c + \varepsilon).$$

This equilibrium is sustainable for $\alpha \in (0, \hat{\alpha})$ and $\delta \in [\underline{\delta}^m(\alpha), 1)$, with $\underline{\delta}^m(\alpha) = \underline{\delta}^e(\alpha) \equiv \frac{l_s - c + \varepsilon}{l_s - c + \varepsilon + \alpha(l_s - c)}$.

In the monitoring-by-rejection equilibrium, the expert makes honest recommendations and charges consumers their loss from the serious problem, which are similar to the experience-service seller's strategy in the optimal equilibrium. Nevertheless, consumers reject the recommendation with a positive probability, which results in undertreatment for the serious problem. As the expert becomes more patient, consumer acceptance rate gradually increases but never reaches full acceptance. As a result, the expert's profit as well as market efficiency gradually increase as the expert becomes more patient but never achieve the first best levels. This stands in sharp contrast with experience goods markets in which honesty and full efficiency are jointly achieved when the seller's discount factor is above a cutoff value.

To understand this equilibrium, first note that although the expert makes honest recommendations on the equilibrium path, it is consumers' best response to sometimes reject the treatment offer because the treatment price is so high that they are just indifferent between accepting and rejecting the treatment recommendation.

To support the monitoring-by-rejection equilibrium, we have to consider both on-schedule and off-schedule deviations by the expert. The expert can make an off-schedule deviation to a price $p' \neq l_s$. We assume that consumers believe the expert will recommend the treatment for both types of problems at $p' \in [c, l_s)$ and for the minor problem at $p' \in [c - \varepsilon, c)$. Given this off-equilibrium belief, consumers will reject p' with probability one, yielding the expert zero profit in the current period. In addition, a price deviation will trigger the punishment phase from next period onwards. So, there is no profitable price deviation.

The expert can also make an on-schedule deviation by recommending the treatment for the minor problem. Since consumers accept the treatment with a positive probability, the on-schedule deviation is not always detected and punished, so the equilibrium involves imperfect public monitoring. Consider that a consumer has the minor problem. If the expert recommends the treatment, he gains a profit $l_s - c + \varepsilon$ in

the current period if his recommendation is accepted. Nevertheless, the expert will lose all future business if his recommendation is rejected and he is caught lying. Consumers' acceptance rate γ^* balances the trade off the expert is facing and makes him just indifferent between recommending and not recommending the treatment for the minor problem. Hence, it is the expert's best response to recommend no treatment for the minor problem. Specifically, the following no-lying condition holds at γ^* :

$$\underbrace{\overbrace{(1 - \gamma^*)}^{\text{Prob of being caught lying}} \times \frac{\delta}{1 - \delta} \overbrace{\alpha(l_s - c)\gamma^*}^{\text{profit per period}}}_{\text{future loss}} = \underbrace{\gamma^*(l_s - c + \varepsilon)}_{\text{current gain from lying}}, \quad (3)$$

yielding $\gamma^* = 1 - \frac{(1-\delta)(l_s-c+\varepsilon)}{\delta\alpha(l_s-c)}$, which is positive for $\delta > \underline{\delta}^m(\alpha)$. Because consumers' acceptance rate is less than one, the serious problem is sometimes left unrepaired, resulting in inefficiency from undertreatment. Note that the expert fully extracts the entire surplus from trade, so the difference between the surplus of the first best and the expert's profit is the efficiency loss from undertreatment.

It can be verified that consumer acceptance rate γ^* increases in δ and α . So, the efficiency of the equilibrium increases when the expert is more patient or the consumer's problem is more likely to result in a substantial loss. If the expert cares more about future business, a smaller consumer rejection rate is sufficient to deter him from cheating. As the discount factor approaches one, γ^* converges to (although never reaching) one and the monitoring-by-rejection equilibrium approaches (although never reaching) full efficiency. Nevertheless, consumers cannot always accept the treatment because full acceptance of the treatment makes it impossible for consumers to monitor expert honesty. As a result, the expert will always recommend the treatment, causing the monitoring-by-rejection equilibrium to collapse. The acceptance rate γ^* also increases in the likelihood of the serious problem (α). This is because the expert's equilibrium profit increases in α , so he bears a larger future loss from lying when α increases and therefore a smaller rejection rate is sufficient to keep him honest.

It is worth noting that the minimum discount factor necessary to sustain the monitoring-by-rejection equilibrium coincides with that required to sustain the honest and efficient equilibrium in experience services markets. This is because the minimum discount factor $\underline{\delta}^m(\alpha)$ is derived when the equilibrium acceptance rate γ^* is zero. Canceling out one γ^* from both sides of the equation (3) and evaluating the simplified equality at $\gamma^* = 0$, this condition coincides with the no lying condition (2) in experience goods markets

when the equality holds. So, honesty can be achieved in both credence goods and experience goods markets when $\delta \geq \underline{\delta}^m(\alpha) = \underline{\delta}^e(\alpha)$, but the first best is not attainable in the former market.

Fong (2005) studies a one-shot game in which there are two treatments, and it is efficient to use the expensive treatment for the serious problem and the inexpensive treatment for the minor problem. In Fong's equilibrium, consumers reject the expensive treatment with a positive probability to keep the expert honest. Although Fong's equilibrium shares some similar features to the monitoring-by-rejection equilibrium, the driving forces for Fong's equilibrium and ours are completely different. Fong considers a short-lived expert, so honesty is driven by current profit consideration, not concerns for future business. In Fong's equilibrium, consumers reject the expensive treatment with a positive probability but accept the inexpensive treatment with probability one. So, when recommending the expensive treatment for the minor problem, the expert faces the trade-off between a higher profit margin and a lower acceptance rate, which balances off his cheating incentives. In contrast, in our setting, because it is inefficient to fix the minor problem, the expert cannot make a positive profit from honestly reporting the minor problem in a static setting and hence will always misreport the minor problem as the serious problem, causing the market in the one-shot game to collapse. As a result of the different driving forces for expert honesty, the degree of the expert's patience plays a crucial role in our equilibrium, but it does not affect the equilibrium in Fong as the expert is short-lived.¹²

The following comparative statics are derived directly from Proposition 1 (proof omitted). These comparative statics are useful for comparing the profit of the monitoring-by-rejection equilibrium with that of the one-price-fix-all equilibrium which will be characterized at the end of this section.

Corollary 1 π^m strictly increases in α and δ ; the cutoff discount factor $\underline{\delta}^m(\alpha)$ strictly decreases in α .

The expert's average profit increases in α and δ because consumers' acceptance rate γ^* increases in these parameters. To see $\underline{\delta}^m(\alpha)$ decreases in α , note that the expert's equilibrium profit is higher when the problem is more likely to be serious. So, the expert bears a larger future loss from lying and therefore is willing to make honest recommendations at a lower discount factor.

¹²To see another difference, we can verify that in our setting consumers' acceptance rate for treatments increases in l_s (i.e., $\partial\gamma^*/\partial l_s > 0$). However, in Fong, the acceptance rate for the minor treatment stays at one when l_s increases but the acceptance rate for the expensive treatment decreases in l_s . So the overall acceptance decreases in l_s .

One-price-fix-all equilibrium We now turn to the one-price-fix-all equilibrium. In this equilibrium, the expert charges a consumer the average loss of her problem and always recommends the treatment. Consumers do not monitor expert honesty. Instead, they expect the expert to always repair their problems at the quoted price even though it is inefficient to repair the minor problem, and the expert's refusal to treat a consumer will trigger the punishment phase.

Define $\tilde{\alpha} \equiv \frac{c - \varepsilon - l_m}{l_s - \varepsilon - l_m}$ and $\underline{\delta}^o(\alpha) \equiv \frac{c - E(l)}{(1 - \alpha)\varepsilon}$. We characterize below the one-price-fix-all equilibrium which is also stationary:

Proposition 2 *In the one-price-fix-all equilibrium, the expert posts $p = E(l)$ for the treatment and recommends the treatment irrespective of the consumer's problem. Consumers accept the expert's treatment with probability one if the expert recommended the treatment to all previous consumers and did not deviate in price in the past. Otherwise, the game reverts to the static Nash equilibrium perpetually. The average profit of the equilibrium is*

$$\pi^o \equiv \alpha(l_s - c) - (1 - \alpha)(c - \varepsilon - l_m).$$

For $\alpha \in (\tilde{\alpha}, \hat{\alpha})$, the one-price-fix-all equilibrium is sustainable if and only if $\delta \in [\underline{\delta}^o(\alpha), 1)$; it is not sustainable for $\alpha \in (0, \tilde{\alpha}]$.

Since $l_m < E(l) < l_s$, consumers are overcharged for repairing the minor problem but receive a positive surplus from fixing the serious problem. Because the consumer's expected benefit from the treatment equals to the price, it is optimal for them to accept the expert's recommendation with probability one. In this equilibrium, the expert is expected to always repair consumers' problems, so he does not have any on-schedule deviation. A price deviation or refusal to treat consumers are off-schedule deviations which are perfectly observed by consumers and will be punished from the next period onward. So, this equilibrium involves perfect public monitoring.

The condition $\tilde{\alpha} < \alpha$ ensures that the equilibrium price $E(l)$ is greater than the average cost of fixing the problem so that the expert can earn a positive expected profit. Nevertheless, given the assumption $E(l) < c$, the price is too low to cover the expert's cost of repairing the serious problem. So, the expert has an incentive to cherry-pick the consumers with the minor problem and refuse to treat those with the

serious problem. When the discount factor is greater than the cutoff $\underline{\delta}^o(\alpha)$, the expert's current gain from rejecting a consumer with the serious problem is outweighed by his loss from losing all future business, and the equilibrium is sustainable.

In the one-price-fix-all equilibrium, consumers always consume the expert's services on the equilibrium path, and yet the expert does not honestly report the loss of their problems and the equilibrium involves inefficiency from overtreatment for the minor problem. Since the expert fully extracts the surplus from trade, the difference between the first best surplus and the expert's profit captures the efficiency loss from overtreatment. The one-price-fix-all equilibrium draws sharp contrast with the optimal equilibrium in experience goods markets wherein full consumption on the equilibrium path facilitates monitoring for quality and induces honest recommendations as well as efficient treatment provision.

Another set of comparative statics useful for comparing the monitoring-by-rejection equilibrium and one-price-fix-all equilibrium follows (proof omitted):

Corollary 2 π^o strictly increases in α and remains constant in δ . When $\alpha \in (\tilde{\alpha}, \hat{\alpha})$, the cutoff discount factor $\underline{\delta}^o(\alpha)$ strictly decreases in α .

Unlike the monitoring-by-rejection equilibrium, the efficiency of the one-price-fix-all equilibrium does not increase in the expert's discount factor, so π^o remains constant in δ . In addition, π^o increases in α because it is efficient to fix the serious problem but inefficient to fix the minor problem. It can be verified that the derivative $\frac{\partial \underline{\delta}^o(\alpha)}{\partial \alpha} = -\frac{L_s - c}{(1-\alpha)^2 \varepsilon} < 0$. This is because when α is higher, the expert can charge a higher price for the treatment, so that he has a smaller gain from dumping costly consumers. In addition, because π^o increases in α , the expert has more future profits to lose from rejecting consumers as α increases. So, it is easier to sustain the one-price-fix-all equilibrium.

In our game, because the expert's recommendation history is publicly observable, the expert and consumers can base their strategies on the entire recommendation history or some summary statistics such as frequency of the treatment recommendation. Nevertheless, the next proposition shows that it is without loss of generality to focus on the monitoring-by-rejection and the one-price-fix-all equilibria because the expert's maximum profit from any PPE is bounded above by the maximum profits from these two equilibria.

Proposition 3 *For a given pair of (α, δ) , with $(\alpha, \delta) \in (0, \hat{\alpha}) \otimes (0, 1)$, the expert's average profit in any Perfect Public equilibrium is bounded above by $\max\{\pi^m, \pi^o\}$.*

To prove Proposition 3, we adopt the approach pioneered by Abreu, Pearce and Stacchetti (1990) to characterize the perfect public equilibrium payoff set for the expert and show that the upper bound of the payoff set is either π^m or π^o . There are five possible public outcomes in our game: 1) the expert recommends no treatment and the consumer suffers a minor loss, 2) the expert recommends no treatment and the consumer suffers a substantial loss, 3) the expert recommends the treatment, the consumer accepts the treatment and receives utility $-p$, 4) the expert recommends the treatment, the consumer rejects it and suffers a minor loss, and 5) the expert recommends the treatment, the consumer rejects it and suffers a substantial loss. The set of public outcomes is denoted by $Y = \{1, 2, 3, 4, 5\}$.

An action profile $\sigma = (p, \beta_m, \beta_s, \gamma)$ determines the probability distribution over the public outcomes $f(y|\sigma)$, $y \in Y$. The action profile $\sigma \in \Sigma$ is enforceable on a payoff set $W \subseteq R^2$ if there exists a mapping $\nu : Y \rightarrow W$ such that for each player i and $\sigma'_i \in \Sigma_i$,

$$\begin{aligned} u_i(\sigma, \nu) &= (1 - \delta)u_i(\sigma) + \delta \sum_{y \in Y} \nu(y) f(y|\sigma) \\ &\geq (1 - \delta)u_i(\sigma'_i, \sigma_{-i}) + \delta \sum_{y \in Y} \nu(y) f(y|\sigma'_i, \sigma_{-i}). \end{aligned}$$

In other words, player i 's action maximizes his/her payoff given other players' actions and the continuation payoff $\nu(y)$. The payoff set W is self generating if for every element $w \in W$, there exists an action profile $\sigma \in \Sigma$, enforced by ν on W , such that $w_i = u_i(\sigma, \nu)$. The set of perfect public equilibrium payoff is the maximum self-generating set. (See Chapters 7 and 8 in Mailath and Samuelson 2006).

In our model, consumers are all short lived and therefore their continuation payoffs are zero. Therefore, we maximize the self-generating set W_i , i being the expert, subject to short-lived consumers' incentive constraint

$$\gamma = \arg \max_{\gamma'} (E(l|\beta_m, \beta_s) - p),$$

and we show that the upper bound of W_i is $\max\{\pi^m, \pi^o\}$. The proof is relegated to Appendix B.

Proposition 3 is important because it allows us to focus on π^m and π^o to pin down the optimal equilibrium

for any parameter configurations. Recall Proposition 2 that when $\alpha \leq \tilde{\alpha}$, the one-price-fix-all equilibrium is not sustainable for any $\delta < 1$ and hence π^m is the highest attainable profit in this parameter range. For $\alpha \in (\tilde{\alpha}, \hat{\alpha})$, both the monitoring-by-rejection and the one-price-fix-all equilibria are sustainable when the discount factor is sufficiently high. In the following analysis, we characterize the condition under which the monitoring-by-rejection equilibrium dominates the one-price-fix-all equilibrium. To begin, we first compare the minimum discount factors necessary to support each type of equilibrium.

Lemma 2 *There exists a unique cutoff likelihood $\bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha})$ (See Figure 1) such that $0 \leq \underline{\delta}^m(\alpha) \leq \underline{\delta}^o(\alpha) \leq 1$ for $\alpha \in (\tilde{\alpha}, \bar{\alpha}]$ and $0 \leq \underline{\delta}^o(\alpha) < \underline{\delta}^m(\alpha) \leq 1$ for $\alpha \in (\bar{\alpha}, \hat{\alpha})$.*

By Corollaries 1 and 2, the cutoff discount factors $\underline{\delta}^m(\alpha)$ and $\underline{\delta}^o(\alpha)$ are both decreasing in α and are illustrated in Figure 1. It is best to understand the comparison by considering the two extreme cases: $\alpha = \tilde{\alpha}$ and $\alpha = \hat{\alpha}$. First, consider $\alpha = \tilde{\alpha}$. Proposition 2 shows that the one-price-fix-for-all equilibrium is not sustainable for all $\delta < 1$ at $\tilde{\alpha}$. By contrast, the monitoring-by-rejection equilibrium is sustainable as long as δ is sufficiently close to one, i.e., $\delta > \underline{\delta}^m(\tilde{\alpha}) \in (0, 1)$. By continuity, $\underline{\delta}^m(\alpha)$ lies below $\underline{\delta}^o(\alpha)$ when α is slightly greater than $\tilde{\alpha}$. As α approaches to $\hat{\alpha}$, it is easier to support the one-price-fix-for-all equilibrium. Note that in Figure 1, $\underline{\delta}^o(\hat{\alpha}) = 0$. This is because when $\alpha = \hat{\alpha}$, consumers' average loss $E(l)$ equals the treatment cost for the serious problem. Hence, it is optimal for the expert to repair the serious problem even when he has no concerns for future business. By contrast, in order to support the monitoring-by-rejection equilibrium, the expert's discount factor must be positive. If the expert has no concerns for future business, he would strictly prefer recommending the treatment for the minor problem because he has a positive gain in the current period from lying.

Both the monitoring-by-rejection and the one-price-fix-all equilibria are sustainable for $\alpha \in (\tilde{\alpha}, \hat{\alpha})$ and $\delta > \max\{\underline{\delta}^o(\alpha), \underline{\delta}^m(\alpha)\}$ (refer to Figure 1). In this case, the most profitable equilibrium depends on the comparison between π^m and π^o . For a fixed α , define $\delta_1(\alpha) \equiv \frac{l_s - c + \varepsilon}{l_s - c + \varepsilon + (1 - \alpha)(c - \varepsilon - l_m)}$ which is solved from $\pi^m = \pi^o$. Next, we characterize the expert's optimal profit for all configurations of (α, δ) with $(\alpha, \delta) \in (0, \hat{\alpha}) \otimes (0, 1)$.

Proposition 4 *There exists a unique cutoff $\alpha^* \in (0, \hat{\alpha})$ such that*

i) $\forall \alpha \in (0, \alpha^*]$, the maximum average profit is π^m for $\delta \in [\underline{\delta}^m(\alpha), 1)$ and zero, otherwise;

ii) $\forall \alpha \in (\alpha^*, \bar{\alpha}]$, the maximum average profit is π^m for $\delta \in [\underline{\delta}^m(\alpha), \underline{\delta}^o(\alpha)] \cup [\delta_1(\alpha), 1)$, π^o for $\delta \in (\underline{\delta}^o(\alpha), \delta_1(\alpha))$ and zero, otherwise;

iii) $\forall \alpha \in (\bar{\alpha}, \hat{\alpha})$, the maximum average profit is π^o for $\delta \in [\underline{\delta}^o(\alpha), \delta_1(\alpha))$, π^m for $\delta \in [\delta_1(\alpha), 1)$ and zero, otherwise.

Figure 1

We illustrated the maximum attainable average profit in Figure 1. The expert's maximum attainable average profit is π^m in the red striped area and is π^o in the blue shaded area with squares. It is zero for the remaining area.

When does the monitoring-by-rejection equilibrium dominate the one-price-fix-all equilibrium? Because the expert extracts the entire surplus from trade in both equilibria, the monitoring-by-rejection equilibrium is more profitable when it is more efficient than one-price-fix-all equilibrium. While the monitoring-by-rejection equilibrium involves undertreatment for the serious problem, the one-price-fix-all equilibrium involves overtreatment for the minor problem. The efficiency comparison depends on the likelihood of the serious problem as well as the expert's discount factor.

When consumers' problems are most likely to be minor (the parameter range $\alpha \in (0, \alpha^*]$), overtreatment for the minor problem is more costly than undertreatment for the serious problem. Consequently, the monitoring-by-rejection equilibrium dominates the one-price-fix-all equilibrium both in efficiency and profit.

Now, consider that the likelihood of the serious problem is relatively high (the parameter range $\alpha \in (\bar{\alpha}, \hat{\alpha})$). When $\delta \geq \delta_1(\alpha)$, $\pi^m \geq \pi^o$. This is because as δ converges to one, consumers' acceptance rate converges to one in the monitoring-by-rejection equilibrium, and hence the expert's profit converges to the first best. By contrast, the minor problem is always inefficiently repaired with probability one in the one-price-fix-all equilibrium, yielding π^o dominated by π^m . As δ keeps decreasing, a larger rejection rate is necessary to discipline the expert to make honest recommendations, and the monitoring-by-rejection equilibrium becomes less efficient. As a result, when δ falls below $\delta_1(\alpha)$, the one-price-fix-all equilibrium dominates the monitoring-by-rejection equilibrium in efficiency and profit.

Finally, when α is in the intermediate range $(\alpha^*, \bar{\alpha}]$, something unusual happens. For high δ , the monitoring-by-rejection equilibrium dominates. As δ falls below $\delta_1(\alpha)$, the one-price-fix-all equilibrium dominates the monitoring-by-rejection equilibrium in efficiency. However, as δ continues to fall below $\underline{\delta}(\alpha)$, the expert is no longer willing to fix the serious problem at a loss, causing the one-price-fix-all equilibrium to collapse. By contrast, the monitoring-by-rejection equilibrium is still sustainable, although giving the expert a lower profit. So the monitoring-by-rejection equilibrium dominates again.

The next proposition investigates the welfare property of the most profitable equilibrium. The proof is relegated to Appendix B.

Proposition 5 *The most profitable equilibrium maximizes the joint surplus.*

Recall that consumer surplus is zero in the most profitable equilibrium. If there exists an equilibrium which generates a larger joint surplus than the most profitable equilibrium, consumers must have a positive surplus in that equilibrium. In the proof we characterize the PPE frontier for any positive consumer surplus and compute the corresponding joint surplus. We show that the joint surplus on the PPE frontier is π^o when consumer surplus is positive. Because the joint surplus of the most profitable equilibrium is $\max\{\pi^o, \pi^m\}$, it is weakly larger than the surplus of any PPE in which consumers have a positive surplus.

We close this section by reiterating the importance of studying credence expert services in a dynamic setting. Although credence goods and experience goods are not distinguishable in a static monopoly setting, the optimal equilibria in the repeated game are very different across these two markets. In the experience

goods monopoly market, the first best can be achieved when the expert's discount factor is greater than a cutoff. By contrast, in the credence goods monopoly market, efficiency gradually increases in the expert's reputation concerns but never achieves the first best for any $\delta < 1$.

Because the monopoly market involves efficiency loss from overtreatment or undertreatment for any $\delta < 1$, we investigate in the next section whether competition in markets for credence goods can improve market efficiency.

5 Competitive market

In this section we consider a market with $n \geq 2$ experts and investigate how competition affects the monitoring technology for honesty and market efficiency. In the monopoly expert market, expert honesty is monitored through consumer rejection, but it comes at the expense of unrepaired serious problem. In a competitive market, consumer rejection is still necessary for the monitoring of expert honesty. Nevertheless, rejection does not need to result in undertreatment because consumers can solicit second opinions after rejecting the first treatment recommendation. When a consumer solicits multiple opinions, she can identify cheating experts through conflicting recommendations. When an expert makes an unnecessary treatment recommendation, he risks losing all future business when the consumer solicits an honest second opinion. We explore this idea in this section.

We make minimal modifications to the monopoly model and retain its key elements, with the exception of adding the following new features. First, consumers pay a small search cost $k > 0$ per visit. The assumption of a positive search cost captures the reality that search in expert markets often involves delay which is costly. Second, we allow entry and exit to maintain a stable pool of experts in the market in each period. When an expert exits the market, he is replaced by a new expert. Allowing entry and exit is realistic and makes our model tractable. Third, following the existing literature,¹³ we assume that an expert cannot identify whether a consumer has visited another expert in the past.

Events in period t unfold in the following sequence: Experts first simultaneously post treatment prices

¹³See Pesendorfer and Wolinsky (2003) and Wolinsky (1993).

which become public information. A consumer arrives in the market and consults an expert. The expert learns the consumer's problem and makes a recommendation. The consumer either follows the expert's recommendation or goes on to search for another opinion. At the end of the period, experts' prices, the consumer's utility and the recommendations made by all the experts she has visited become public information.

Next, we identify a class of equilibria in which consumers actively search for second opinions and use second opinions to monitor expert honesty. We therefore call these equilibria in the following proposition “*search equilibria*”.

Proposition 6 *When k is low, there exists a continuum of stationary search equilibria indexed by p . In each period, experts post the same price $p \in (c, c + k]$. They recommend the treatment for the serious problem with probability one and for the minor problem with probability $\beta_m^*(p; k) \in (0, 1)$. A consumer randomly visits an expert when arriving at the marketplace. If recommended the treatment on her first visit, the consumer accepts it with probability $\gamma^*(p; k, \delta, n) \in (0, 1)$ and searches for a second opinion with the complementary probability. If recommended the treatment again on her second visit, the consumer accepts the recommendation with probability one. Whenever the consumer is recommended no treatment, she exits the market. When the consumer receives conflicting recommendations, the expert who recommends the treatment loses all future business and exits the market. This equilibrium is sustainable for $\delta \geq \underline{\delta}(p; k, n) \equiv \frac{n(p-c+\varepsilon)\beta_m^*(p; k)}{n(p-c+\varepsilon)\beta_m^*(p; k) + \alpha(1-\beta_m^*(p; k))(p-c)}$.*

To understand a search equilibrium indexed by p , we first analyze a consumer's incentives to solicit second opinions. When all of the experts lie with positive probabilities, the consumer may receive different recommendations from different experts and hence has incentives to search for honest second opinions. If the consumer accepts the treatment on her first visit, her problem will be fixed and she receives a payoff $-p$. Alternatively, the consumer can pay the search cost to solicit a second opinion. If the second opinion recommends no treatment, the consumer infers that she must have the minor problem and hence will choose not to repair it. So, the consumer has a net benefit of $p - l_m$ from search when the second opinion contradicts the first opinion. For the consumer to randomize between accepting the first treatment recommendation and soliciting a second opinion, it requires that the consumer's expected net benefit from search equals her search

cost k , which gives

$$\Pr(\emptyset|p)(p - l_m) = k, \quad (4)$$

where $\Pr(\emptyset|p)$ is the probability that the second opinion recommends no treatment conditional on the first opinion recommending the treatment. When (4) holds, the consumer strictly prefers accepting the second treatment recommendation to soliciting a third opinion. This is because the likelihood that the third opinion recommends no treatment conditional on two treatment recommendations is smaller than $\Pr(\emptyset|p)$, and hence the consumer's expected net benefit from search is strictly less than the search cost.

For the consumer to be willing to enter the market, the following participation constraint must hold

$$p + k \leq \Pr(l_s|p)l_s + \Pr(l_m|p)l_m, \quad (5)$$

where $\Pr(l_i|p)$, $i = m, s$, is the probability that the consumer's problem is i conditional on the first treatment recommendation. If the second opinion also recommends the treatment, the consumer updates her belief of having the serious problem upward and expects a greater loss from the problem. So, the participation constraint (5) implies that the consumer strictly prefers to accept the treatment on her second visit.

Now, we turn to analyze experts' recommendation strategies. An expert is indifferent between whether or not to recommend the treatment to the minor problem when the following condition holds:

$$\delta V \underbrace{\underbrace{(1 - \Pr(e|l_m))}_{\text{consumers on their first visits}} \underbrace{(1 - \gamma)(1 - \beta_m)}_{\text{the consumer receives an honest 2}^{nd} \text{ opinion}}}_{\text{future loss}} = \underbrace{(p - c + \varepsilon) \underbrace{[\Pr(e|l_m) + (1 - \Pr(e|l_m))\gamma]}_{\text{prob of acceptance of the treatment}}}_{\text{current gain}}, \quad (6)$$

where $\Pr(e|l_m) \equiv \frac{\beta_m(1 - \gamma)}{1 + \beta_m(1 - \gamma)}$ is the probability that the consumer is on her second visit conditional on her problem being minor and V is the expert's continuation profit when he is active in the market. When an expert recommends the treatment for the minor problem, the treatment is accepted if the consumer is on her second visit or if she is on her first visit and chooses to accept the treatment, which happens with probability γ . The expert's profit margin from fixing the minor problem is $p - c + \varepsilon$. So, his expected current gain from recommending the treatment for the minor problem is on the right-hand side of (6). On the other hand, the expert risks losing all future business if he is caught lying. This happens when the consumer is on her first visit and decides to solicit a second opinion, and the second opinion happens to be honest. The expert's

discounted expected future loss from lying is therefore given by the left-hand side of (6). When (6) holds, it is the expert's best response to randomize between recommending and not recommending treatment for the minor problem.

Finally, we construct consumers' off-equilibrium beliefs to prevent a price deviation. We assume consumers believe that an expert will recommend the treatment to both types of problems if he deviates to a price $p' > c$ and will recommend the treatment only to the minor problem for $p' \leq c$. Given this off-equilibrium belief, a consumer will not be attracted to the deviating expert on her first visit. The expert can potentially offer a low price to attract consumers who search for second opinions, but $p \leq c + k$ prevents such a price deviation. To see this, given the off-equilibrium belief, consumers will visit the deviant for a second opinion only if $c < p'$. Since the consumer expects the deviant to always recommend the treatment at p' , she prefers to seek treatment from the deviant rather than accept the first treatment recommendation if and only if $p' < p - k$. However, since $p \leq c + k$, $p' < c$, which contradicts $c < p'$. So, for $p \in (c, c + k]$, there does not exist a price deviation which can profitably attract consumers.

For a given price $p \in (c, c + k]$, there exists a search equilibrium which satisfies conditions (4), (5) and (6) when k is sufficiently low and $\delta \geq \underline{\delta}(p; k, n)$. To see that the discount factor must be high enough to sustain the search equilibrium, suppose δ is zero. Then, the expert does not bear any future loss from lying. Nevertheless, he has a positive current gain from lying because there is a positive probability that the consumer is on her second visit and will accept the treatment recommendation with probability one. So, the expert will lie with probability one when he does not care much about future business.

There are two notable properties of the search equilibrium. First, consumer search can eliminate undertreatment but overtreatment still arises when both the first and the second opinions exaggerate the consumer's condition. Second, contrary to the conventional wisdom that second opinions improve honesty,¹⁴ we find the availability of second opinions may induce more dishonest recommendations. Recall that the monopoly expert makes honest recommendations in the monitoring-by-rejection equilibrium. By contrast, the search equilibrium involves small but pervasive cheating among experts. This is because some cheating is

¹⁴To give an example, Wolinsky (1995) argued that "the basic force that imitates experts' incentive to misrepresent minor treatment as major ones is customers' search for multiple opinions."

necessary to induce consumers to search. In other words, full honesty is incompatible with search of second opinions: If experts all make honest recommendations, consumers will not pay the search cost to sample more opinions.

We conduct the comparative statics of the search equilibrium with respect to δ , k and n .

Lemma 3 *Consider an equilibrium indexed by $p \in (c, c+k)$. i) Both $\gamma^*(p; k, \delta, n)$ and experts' profits increase in δ . ii) When k decreases, $\beta_m^*(p; k)$ and $\underline{\delta}(p; k, n)$ decrease, $\gamma^*(p; k, \delta, n)$ increases, and experts' profits stay constant. iii) When n increases, $\beta_m^*(p; k)$ stays constant, $\underline{\delta}(p; k, n)$ increases, $\gamma^*(p; k, \delta, n)$ decreases, and experts' profits also decrease.*

Lemma 3 says that consumers solicit second opinions less often when experts are more patient, yielding experts higher profits. The intuition is the same as in the monopoly market. When experts care more about future business, a smaller rejection rate is sufficient to keep them indifferent between whether or not to recommend the treatment for the minor problem.

Conventional wisdom believes that a market becomes more competitive when the search cost is reduced or the number of sellers increases. Lemma 3 shows these two instruments to influence the competitiveness of the expert market have very different implications on honesty, search frequency and efficiency. Specifically, when k is reduced, experts become more honest, consumers solicit second opinions less frequently, and it is easier to sustain the equilibrium. The equilibrium becomes more efficient because overtreatment is reduced and consumers incur less costly search. To see why experts become more honest, note that as k diminishes, a consumer's gain from soliciting a second opinion must be reduced proportionally for condition (4) to hold. So, the probability that the second opinion contradicts the first opinion must be reduced, which requires the cheating probability $\beta_m^*(p; k)$ to decrease. When other experts become more honest, it is riskier for an expert to lie about the consumer's condition. As a result, the equilibrium is sustainable even when the consumer accepts the expert's recommendation more often. Our analysis shows that policies aiming to reduce consumers' search costs are likely to improve welfare.

When the number of experts in the market increases, it is more difficult to sustain the search equilibrium, consumers will solicit second opinions more frequently, and the honesty of experts' recommendation remain

unchanged. Consequently, the search equilibrium becomes less efficient because consumers incur more costly search. To see the intuition, note that an expert's current gain from making an unnecessary treatment recommendation does not depend on the number of experts. However, the expert has less to lose from cheating when he has more competitors because the expert has a smaller chance of serving consumers in the future. Consequently, it is harder to sustain the search equilibrium as n increases. Moreover, consumers search more often in equilibrium when n increases. Because the expert has less to lose from cheating, his gain from cheating must be reduced proportionally for (6) to hold, and this requires consumers to reject the first treatment recommendation and solicit second opinions more often. The policy implication is that lowering the entry barrier to markets for credence goods reduce efficiencies and could even impede trade because it is harder to sustain the search equilibria.

In a one-shot game, Wolinsky (1993) and Dulleck and Kerschbamer (2006) identify a class of equilibria in which experts randomize between whether or not to lie and consumers randomize between whether or not to seek second opinions. Despite the similar equilibrium behavior in their models and in ours, the driving forces for the equilibria are very different,¹⁵ yielding important differences in comparative statics. Because we want to focus on to what extent reputation concern can affect expert conduct and efficiency, we adopt the assumption that there is only one treatment which is efficient to use for the serious problem but not for the minor problem. Under this assumption, the search equilibrium collapses for short-lived experts. Our study shows that experts' reputation concern and the number of experts have important influence on search frequency, honesty and efficiency. Wolinsky (1993) and Dulleck and Kerschbamer (2006) do not consider reputation mechanism and study short-lived experts. They assume that it is efficient to fix both types of problems. Under their assumption, the number of experts has no influence in expert equilibrium conduct or market efficiency.

¹⁵In Wolinsky (1993) and Dulleck and Kerschbamer (2006), short-lived experts have two treatments and it is efficient to use the expensive treatment for the serious problem and the inexpensive treatment for the minor problem. When deciding whether to recommend the expensive treatment for the minor problem, an expert trades off a high profit margin against a low acceptance rate because the expensive treatment is sometimes rejected and the inexpensive treatment is always accepted. By contrast, in our setting, experts have only one treatment and it is a dominant strategy for them to recommend the treatment for the minor problem in a static setting. Hence, the search equilibrium is not sustainable for short-lived experts. A long-lived expert trades off a current gain from recommending unnecessary services against a future loss of business. Because an expert's degree of patience and the number of experts in each period affect the value of the expert's future business, they also affect consumers' equilibrium search frequency $\gamma^*(p; k, \delta, n)$ as in Corollary 3

Next, we investigate how search equilibria change in the search cost at the limit. Let $\bar{p} \equiv c + k$ denote the upper bound of the price of a search equilibrium.

Proposition 7 *For a given (δ, n) , when $k \rightarrow 0$, $\bar{p} \rightarrow c$, $\beta_m^*(p; k) \rightarrow 0$, $\gamma^*(p; k, \delta, n) \rightarrow 0$, $\underline{\delta}(p; k, n) \rightarrow \underline{\delta}^c(\alpha, n) \equiv \frac{n\alpha\varepsilon}{n\alpha\varepsilon + \alpha(1-\alpha)(c-l_m)}$, and the surplus of the search equilibrium converges to the first best $\alpha(l_s - c)$.*

The limiting case with respect to $\beta_m^*(p; k)$ directly follows from Lemma 3. When $\bar{p} \rightarrow c$, an expert's continuation payoff in the market converges to zero. So, the expert has almost nothing to lose from exiting the market. In order to keep him still in balance between whether or not to cheat, consumers' acceptance rate $\gamma^*(p; k, \delta, n)$ must converge to zero, which minimizes the expert's current gain from cheating and maximizes the probability that he is found out cheating. The cutoff discount factor $\underline{\delta}(p; k, n)$ is affected by two opposing forces when k decreases. On the one hand, an expert's continuation profit V converges to zero as the price converges to c , so he has less to lose when losing future business. This increases $\underline{\delta}(p; k, n)$. On the other hand, the comparative statics $\gamma^*(p; k, \delta, n) \rightarrow 0$ and $\beta_m^*(p; k) \rightarrow 0$ suggests that cheating will be caught almost surely because the probability that a consumer will solicit an honest second opinion converges to one. This reduces $\underline{\delta}(p; k, n)$. In the limit, the minimum discount factor necessary to support the search equilibrium converges to $\underline{\delta}^c(\alpha, n)$.

Proposition 7 implies that competition improves efficiency when the search cost is sufficiently low. While the optimal monopoly equilibrium converges to the first best only when δ converges to 1, the search equilibrium converges to the first best as the search cost converges to zero even when the discount factor is bounded away by 1. In our model, the efficiency gain from competition is due to the use of a more efficient monitoring technology for honesty. In the monopoly market, the serious problem must be left unrepaired sometimes for consumers to monitor expert honesty and the unrepaired serious problem can result in a significant loss of efficiency. By contrast, this inefficiency from unrepaired serious problem is avoided in the search equilibrium. A skeptical consumer searches for second opinions instead of bearing the loss of her problem, so the serious problem is always repaired.

Finally, there are some interesting differences between the role of competition in credence service and

experience service markets in the dynamic setting. In credence service markets, competition improves efficiency through consumer search which provides a more efficient monitoring technology for honesty. In experience service markets, competition does not improve the efficiency of the monitoring technology. In both monopoly and competitive experience service markets, consumers can always learn the honesty of an expert's recommendation through accepting his recommendation. In addition, consumers do not search for second opinions in experience service markets. Instead, they may search for cheaper prices and the price competition makes it harder to sustain the first best than the monopoly experience service market.

To see this, consider the experience service markets with n experts. Suppose that there exists a symmetric equilibrium in which experts post the same price p and make the honest recommendation. Similar to (1), the following IC constraint for truth telling must hold

$$\frac{\delta}{1-\delta} \frac{\alpha(p-c)}{n} \geq p - c + \varepsilon, \quad (7)$$

which requires $\delta \geq \underline{\delta}^{ec}(p) \equiv \frac{n(p-c+\varepsilon)}{n(p-c+\varepsilon)+\alpha(p-c)}$.

Moreover, we need an upper bound on p to prevent experts from price cut. Suppose that consumers' off-equilibrium belief following a price deviation p' is that the deviating expert will always recommend the treatment for $p' \geq c$ and will recommend the treatment only to the minor problem for $p' < c$. Given this off-equilibrium belief, a consumer will not visit the deviant on her first trip. However, if the price differential $p - p'$ is larger than k , it is optimal for the consumer to first visit an expert who posts the equilibrium price p , and then buys the service from the deviant if the first expert recommends the service. This is because the consumer can learn her valuation for the good from the first expert's recommendation and then shops for a cheaper price. Given this consumer behavior, experts may have a profitable deviation from undercutting the price to attract consumers who are shopping for cheaper services. To prevent such a price deviation, the equilibrium price must be bounded above by $c + k$ so that a price cut by k will lead to a negative profit from serving consumers. Since $\underline{\delta}^{ec}(p)$ increases in n but decreases in p , with $p < c + k < l_s$, it is harder to sustain the first best than in the monopoly experience market.

6 Discussion

In the main model, we focus on the parameter range $E(l) < c$ which implies $\alpha < \hat{\alpha} \equiv \frac{c - l_m}{l_s - l_m}$. In this section, we discuss how the results change when $E(l) \geq c$, i.e. $\alpha \geq \hat{\alpha}$.

Lemma 4 *When $\alpha \geq \hat{\alpha}$, the one-shot game has a unique subgame perfect Nash equilibrium not involving weakly dominated strategies. In the equilibrium, the expert posts the price $p = E(l)$ and always recommends the treatment. Consumers accept the treatment recommendation with probability one.*

In the one-shot game, each possible price is followed by a proper subgame. In subgames following $p \geq c$, it is the expert's weakly dominant strategy to always recommend the treatment. Given that the expert will always recommend the treatment, it is the consumer's best response to accept the recommendation as long as $p \leq E(l)$. In subgames following $p < c$, the consumer will reject the expert's recommendation for the same argument outlined for Lemma 1. Hence, Lemma 4 is the unique SPNE not involving weakly dominated strategies. The static equilibrium involves *overtreatment* for the minor problem.

Lemmas 1 and 4 suggest that trade collapses in the one-shot game if and only if the likelihood of the minor problem is sufficiently high. This is consistent with the equilibrium in the textbook lemon market model, which shows that market collapses when the proportion of "lemon" is high enough. In our model, the minor problem is the "lemon" whereas the serious problem is the "peach".

Next, we argue that the main findings of our analysis continue to hold qualitatively when $E(l) \geq c$. Similar as in Section 3, the static equilibrium in Lemma 4 holds both for credence goods and experience goods because the consumer's post consumption experience does not play any role in the one-shot game. When $\alpha \geq \hat{\alpha}$, the static equilibrium resembles the one-price-fix-all equilibrium in Proposition 2. So, the one-price-fix-all equilibrium holds in the repeated game for any $\delta \in [0, 1)$. How about the monitoring-by-rejection equilibrium? It continues to exist when the expert is sufficiently patient, but the minimum discount factor required to sustain the equilibrium is higher than the case of $\alpha < \hat{\alpha}$. This is because the static Nash equilibrium is more profitable than the static equilibrium under the assumption $\alpha < \hat{\alpha}$ and therefore the expert has less to lose when caught cheating, which makes it harder to sustain the equilibrium. Nevertheless,

when δ is sufficiently close to one, the monitoring-by-rejection equilibrium approaches the first best. Finally, for the competitive setting, the search equilibrium continues to exist when δ is sufficiently high and k is sufficiently low because the equilibrium strategy does not depend on the assumption of $E(l) < c$.

7 Concluding remarks

This paper studies trust building in a dynamic setting of credence goods markets. Although credence goods and experience goods are indistinguishable in static settings, the trust building mechanisms in these two markets are very different in dynamic settings. In credence goods markets, consumers monitor expert honesty through rejection of his services. This stands in sharp contrast with experience service markets in which consumers learn and monitor the quality of the experience services by consuming the services.

In the monopoly credence goods markets, the optimal equilibrium is inefficient for any discount factor less than one; it either involves undertreatment for the serious problem or overtreatment for the minor problem. The efficiency of the optimal equilibrium gradually increases in the expert's discount factor but never achieves the first best. This contrasts sharply with experience goods markets in which the first best is achieved when the seller's discount factor is greater than a threshold.

Competition enhances the reputation mechanism in credence goods markets through consumer search for second opinions. The availability of second opinions provides a more efficient way for consumers to monitor expert honesty. When the search cost is small, competition improves market efficiency, but the efficiency gain comes at the expense of honesty. Our study suggests that reduction of search costs and encouragement of consumers' solicitation of second opinions should be an integral part of any regulatory policies aiming at enhancing welfare.

Appendix A

Proof for Lemma 1: We confine the analysis to the case $p \geq c - \varepsilon$ because it is weakly dominated for the expert to recommend a treatment at $p < c - \varepsilon$. Suppose that trade happens with a positive probability, then p must be at least c . To see this, if $p < c$, the expert will recommend the treatment only when the consumer has the minor problem. Since $l_m < c - \varepsilon \leq p$, the consumer's best response is to reject p with

probability one. Now, consider $c \leq p$. If p is accepted with a positive probability, the expert will recommend p for the minor problem with probability one. This implies $E(l|p) \leq E(l)$, where $E(l|p)$ is the consumer's expected loss conditional on the treatment recommendation. But then it is the consumer's best response to reject p with probability one because $E(l|p) \leq E(l) < c \leq p$. A contradiction. Q.E.D.

Proof for Proposition 1: Given the expert's strategy, a consumer's expected loss from her problem is l_s upon the treatment recommendation. So, the consumer is indifferent between accepting and rejecting the treatment, and therefore it is her best response to accept the treatment with probability γ^* .

Given price $p = l_s$, the expert does not have a profitable deviation in his recommendation strategy. First, consider that the consumer has the serious problem. The expert makes a positive profit from recommending the treatment but makes zero profit from recommending no treatment. It is the expert's best response to recommend the treatment for the serious problem. Suppose that the consumer has the minor problem. Given condition (3), the expert is indifferent between whether or not to recommend the treatment. So, it is the expert's best response to recommend no treatment. Last, for $\gamma^* \geq 0$, it is necessary to have $\delta \geq \underline{\delta}^m$.

Finally, the expert does not have a profitable price deviation. A price deviation outside the range $[c - \varepsilon, l_s)$ is not profitable because the expert will make a loss from repairing a problem at $p' < c - \varepsilon$ and the consumer will reject a price $p' > l_s$, irrespective of her belief about her problem. A price deviation in the range of $[c - \varepsilon, l_s)$ will be rejected given the consumer's off-equilibrium belief specified for Proposition 1 in the main text. Hence, any price deviation leads to zero profit because the expert's recommendation will be rejected in the current period and the deviation will trigger the reversion to the punishment path from the next period onward. Q.E.D.

Proof for Proposition 2: Given the expert's equilibrium strategy, consumers' maximum willingness to pay for the treatment is $E(l)$. Hence, it is consumers' best response to accept the expert's treatment recommendation with probability one.

Next, we show that the expert does not have a profitable deviation. First, consider that the consumer has the serious problem. The expert makes zero profit from refusal to fix the problem because the game will

revert to the static Nash equilibrium from next period onward. So, the no deviation condition requires

$$\begin{aligned} E(l) - c + \frac{\delta\pi^o}{1-\delta} &\geq 0 \\ \delta &\geq \underline{\delta}^o \equiv \frac{c - E(l)}{(1-\alpha)\varepsilon}. \end{aligned} \quad (8)$$

The cutoff $\underline{\delta}^o$ is positive given the assumption $E(l) < c$. It can be verified that $\underline{\delta}^o < 1$ if and only if $\alpha \geq \tilde{\alpha}$.

Now, consider that the consumer has the minor problem. The expert's no deviation constraint is

$$E(l) - c + \varepsilon + \frac{\delta\pi^o}{1-\delta} \geq 0. \quad (9)$$

Condition (8) implies (9) because it is less costly to repair the minor problem than the serious problem. In summary, the expert does not have a profitable deviation in his recommendation strategy when $\delta \geq \underline{\delta}^o$.

Finally, assume that consumers hold the same off-equilibrium belief following a price deviation as in the proof for Proposition 1. Given this off-equilibrium belief, a price different from $E(l)$ will be rejected in the current period and result in zero future profit. Thus, the expert does not have a profitable deviation in price. Q.E.D.

Proof for Lemma 2: Take the difference between $\underline{\delta}^o(\alpha)$ and $\underline{\delta}^m(\alpha)$ defined in Propositions 2 and 1, respectively:

$$\underline{\delta}^o(\alpha) - \underline{\delta}^m(\alpha) = \frac{-\alpha^2(l_s - l_m)(l_s - c) - \alpha[(c - \varepsilon - l_m)\varepsilon + (l_s - c)^2] + (c - \varepsilon - l_m)(l_s - c + \varepsilon)}{(1 - \alpha)[\varepsilon + (1 + \alpha)(l_s - c)]\varepsilon}.$$

Since the denominator is positive, the sign of $\underline{\delta}^o(\alpha) - \underline{\delta}^m(\alpha)$ is determined by the numerator which we denote by $f(\alpha)$. The derivative of $f(\alpha)$ is

$$f'(\alpha) = -2\alpha(l_s - l_m)(l_s - c) - [(c - \varepsilon - l_m)\varepsilon + (l_s - c)^2] < 0.$$

So, $f(\alpha)$ is strictly decreasing in the range of $(\tilde{\alpha}, \hat{\alpha})$.

Next, we show $f(\tilde{\alpha}) > 0$. To see this, note that $\underline{\delta}^o(\tilde{\alpha}) = 1$ and $\underline{\delta}^m(0) = 1$. By Corollary 1, $\underline{\delta}^m(\alpha)$ is strictly decreasing in α , and hence $\underline{\delta}^m(\tilde{\alpha}) < \underline{\delta}^o(\tilde{\alpha}) = 1$.

Now, we evaluate $f(\alpha)$ at $\alpha = \hat{\alpha} \equiv \frac{c - l_m}{l_s - l_m}$ and obtain $f(\hat{\alpha}) = \frac{-(l_s - c)(l_s - c + \varepsilon)\varepsilon}{l_s - l_m} < 0$. Given that $f(\alpha)$ is continuous and decreasing and that $f(\tilde{\alpha}) > 0$ and $f(\hat{\alpha}) < 0$, there exists a unique $\bar{\alpha} \in (\tilde{\alpha}, \hat{\alpha})$, such that $\underline{\delta}^o \geq \underline{\delta}^m$ for $\alpha \in (\tilde{\alpha}, \bar{\alpha}]$ and $\underline{\delta}^o < \underline{\delta}^m$ for $\alpha \in (\bar{\alpha}, \hat{\alpha})$. Q.E.D.

Proof for Proposition 4: The proof is divided into 4 steps. Step 1 defines α^* and characterizes properties of α^* and $\delta_1(\alpha)$, which will be used for the subsequent steps. Steps 2, 3 and 4 prove cases i), ii) and iii) in the Proposition, respectively. Figure 1 will facilitate the understanding of the proof.

Step 1. Define α^* as the value which solves $\delta_1(\alpha) = \underline{\delta}^o(\alpha)$. We show that there exists a unique $\alpha^* \in (\tilde{\alpha}, \bar{\alpha})$. To begin, we first prove that there exists a unique $\alpha^* \in (\tilde{\alpha}, \hat{\alpha})$. Evaluate $\delta_1(\alpha)$ and $\underline{\delta}^o(\alpha)$ at $\alpha = \tilde{\alpha}$ and $\alpha = \hat{\alpha}$, respectively. We have

$$\delta_1(\tilde{\alpha}) = \frac{l_s - c + \varepsilon}{l_s - c + \varepsilon + (1 - \tilde{\alpha})(c - \varepsilon - l_m)} < 1 = \underline{\delta}^o(\tilde{\alpha}) \quad (10)$$

$$\delta_1(\hat{\alpha}) = \frac{l_s - c + \varepsilon}{l_s - c + \varepsilon + (1 - \hat{\alpha})(c - \varepsilon - l_m)} > 0 = \underline{\delta}^o(\hat{\alpha}). \quad (11)$$

Note that $\delta_1(\alpha)$ and $\underline{\delta}^o(\alpha)$ are continuous in the range of $(\tilde{\alpha}, \hat{\alpha})$ and $\delta_1(\alpha)$ is strictly increasing while $\underline{\delta}^o(\alpha)$ is strictly decreasing (Corollary 2). As a result, conditions (10) and (11) implies that there is a unique $\alpha^* \in (\tilde{\alpha}, \hat{\alpha})$ such that $\delta_1(\alpha) > \underline{\delta}^o(\alpha)$ for $\alpha > \alpha^*$ and $\delta_1(\alpha) \leq \underline{\delta}^o(\alpha)$ for $\alpha \leq \alpha^*$.

Next, we show $\alpha^* < \bar{\alpha}$ by contradiction. Suppose $\alpha^* \geq \bar{\alpha}$. Then, it follows that

$$\underline{\delta}^m(\bar{\alpha}) = \underline{\delta}^o(\bar{\alpha}) \geq \underline{\delta}^o(\alpha^*) = \delta_1(\alpha^*) \geq \delta_1(\bar{\alpha}). \quad (12)$$

The equalities follows from the definitions of $\bar{\alpha}$ and α^* , respectively. The first inequality holds because $\underline{\delta}^o(\alpha)$ strictly decreases in α and the second inequality holds because $\delta_1(\alpha)$ strictly increases in α . Furthermore, it can be verified that $\underline{\delta}^m(\tilde{\alpha}) = \delta_1(\tilde{\alpha})$. Since $\underline{\delta}^m(\alpha)$ strictly decreases in α while $\delta_1(\alpha)$ strictly increases in α , $\underline{\delta}^m(\alpha) < \delta_1(\alpha)$ for all $\alpha > \tilde{\alpha}$, which contradicts (12) given $\bar{\alpha} > \tilde{\alpha}$.

Step 2. Consider $\alpha \in (0, \alpha^*]$. By Step 1, $\alpha^* < \bar{\alpha}$, so $\underline{\delta}^m(\alpha) < \underline{\delta}^o(\alpha)$ for $\alpha \in (0, \alpha^*]$. The *one-price-fix-for-all* equilibrium is not sustainable if $\underline{\delta}^m(\alpha) \leq \delta < \underline{\delta}^o(\alpha)$ by Proposition 2. So, the *monitoring-by-rejection* equilibrium yields the expert the highest profit. Next, consider $\underline{\delta}^o(\alpha) \leq \delta < 1$. By Step 1, $\delta_1(\alpha) \leq \underline{\delta}^o(\alpha)$ for $\alpha \leq \alpha^*$. Consequently, $\pi^o \leq \pi^m$ for $\delta_1(\alpha) \leq \underline{\delta}^o(\alpha) \leq \delta$ by the definition of $\delta_1(\alpha)$.

Step 3. Consider $\alpha \in (\alpha^*, \bar{\alpha}]$. By Lemma 2, $\underline{\delta}^m(\alpha) < \underline{\delta}^o(\alpha)$ for $\alpha < \bar{\alpha}$ and therefore the maximum average profit is π^m for $\underline{\delta}^m(\alpha) \leq \delta \leq \underline{\delta}^o(\alpha)$. Now, consider $\underline{\delta}^o(\alpha) < \delta$. By Step 1, $\delta_1(\alpha) > \underline{\delta}^o(\alpha)$ for $\alpha \in (\alpha^*, \bar{\alpha}]$. It follows that $\pi^m \leq \pi^o$ for $\underline{\delta}^o(\alpha) \leq \delta < \delta_1(\alpha)$ and $\pi^m > \pi^o$ for $\delta_1(\alpha) \leq \delta < 1$.

Step 4. Consider $\alpha \in (\bar{\alpha}, \hat{\alpha}]$. By Lemma 2, $\underline{\delta}^o(\alpha) < \underline{\delta}^m(\alpha)$ for $\bar{\alpha} < \alpha$. So, the maximum average profit is

π^o for $\underline{\delta}^o(\alpha) \leq \delta < \underline{\delta}^m(\alpha)$. Both π^m and π^o can be supported for $\underline{\delta}^m(\alpha) \leq \delta < 1$. Since $\underline{\delta}^m(\alpha) < \delta_1(\alpha)$ for $\alpha \in (\bar{\alpha}, \hat{\alpha})$, by Step 1, $\pi^m \geq \pi^o$ for $\delta_1(\alpha) \leq \delta < 1$ and $\pi^m < \pi^o$ for $\underline{\delta}^m(\alpha) \leq \delta < \delta_1(\alpha)$. Q.E.D.

Proof for Proposition 6: Define

$$\beta_m^*(p; k) \equiv \frac{(1 - \alpha)(p - l_m - k) - \sqrt{(1 - \alpha)^2(p - l_m - k)^2 - 4k\alpha(1 - \alpha)(p - l_m)}}{2(1 - \alpha)(p - l_m)} \quad (13)$$

$$\underline{\delta}(p; k, n) \equiv \frac{n(p - c + \varepsilon)\beta_m^*(p; k)}{n(p - c + \varepsilon)\beta_m^*(p; k) + \alpha(1 - \beta_m^*(p; k))(p - c)}, \quad (14)$$

$$\gamma^*(p; k, n) \equiv 1 - \frac{n(1 - \delta)(p - c + \varepsilon)}{n(1 - \delta)(p - c + \varepsilon)(1 - \beta_m^*(p; k)) + \delta\alpha(1 - \beta_m^*(p; k))(p - c)}. \quad (15)$$

First, note that the expression under the square root in (13) is positive if $k \leq \frac{(c - l_m)(1 - \sqrt{\alpha})}{1 + \sqrt{\alpha}}$. So, $\beta_m^*(p; k) \in [0, 1]$ for $k \leq \frac{(c - l_m)(1 - \sqrt{\alpha})}{1 + \sqrt{\alpha}}$.

Next, we show that consumers' strategy is a best response given experts' strategies. When a consumer receives a treatment recommendation on her first visit, the probability that the second opinion recommends no treatment is $\Pr(\emptyset|p) = \frac{(1 - \alpha)\beta_m(1 - \beta_m)}{\alpha + (1 - \alpha)\beta_m}$. Substitute $\Pr(\emptyset|p)$ into the search condition (4), it becomes

$$\frac{(1 - \alpha)\beta_m(1 - \beta_m)(p - l_m)}{\alpha + (1 - \alpha)\beta_m} = k. \quad (16)$$

It can be verified that (16) is satisfied at $\beta_m^*(p; k)$ defined in (13). Note that the item in the square root in $\beta_m^*(p; k)$ is positive and $\beta_m^*(p; k) \in (0, 1)$ when k is sufficiently low.

Next, consider the consumer's participation constraint (5). Substitute $\Pr(l_s|p) = \frac{\alpha}{\alpha + (1 - \alpha)\beta_m}$ and $\Pr(l_m|p) = \frac{(1 - \alpha)\beta_m}{\alpha + (1 - \alpha)\beta_m}$ into (5), we obtain

$$p + k \leq \frac{\alpha l_s + (1 - \alpha)\beta_m l_m}{\alpha + (1 - \alpha)\beta_m}. \quad (17)$$

When k converges to 0, $\beta_m^*(p; k)$ converges to 0. So, the right hand side of (17) converges to l_s . Since $p \leq c + k$ and $c < l_s$, the participation constraint (17) is satisfied when k is sufficiently small. We conclude that given p and $\beta_m^*(p; k)$, it is the consumer's best response to randomize between accepting the treatment on her first visit and searching for a second opinion.

Next, we show that it is the consumer's best response to accept the second treatment recommendation with probability one. The probability that the third opinion recommends no treatment conditional on the first two opinions recommending the treatment is denoted by $\Pr(\emptyset|pp) = \frac{(1 - \alpha)(\beta_m)^2(1 - \beta_m)}{\alpha + (1 - \alpha)(\beta_m)^2}$. Because

$\Pr(\emptyset|pp) < \Pr(\emptyset|p)$, it follows that $\Pr(\emptyset|pp)(p - l_m) < k$ and hence the consumer strictly prefers accepting the second treatment recommendation to searching for the third opinion. Finally, the participation constraint (17) is satisfied when the consumer receives the second treatment recommendation. Hence, it is the consumer's best response to accept the second treatment recommendation with probability one.

Now, we show that experts' strategies are their best response. We show that experts have no profitable deviation in their recommendation strategies. When the expert diagnoses that a consumer has the minor problem, the probability that she is on her second visit is

$$\Pr(e|l_m) = \frac{(1 - \alpha)\frac{1}{n}\beta_m(1 - \gamma)}{(1 - \alpha)[\frac{1}{n} + \frac{1}{n}\beta_m(1 - \gamma)]} = \frac{\beta_m(1 - \gamma)}{1 + \beta_m(1 - \gamma)}. \quad (18)$$

Denote by $\Pi \equiv \alpha(p - c) + (1 - \alpha)(p - c + \varepsilon)(\gamma\beta_m + (1 - \gamma)(\beta_m)^2)$ the industry profit in a period. The expected industry profit from treating the serious problem is $\alpha(p - c)$ because the serious problem is repaired with probability one. The minor problem is fixed with probability $\gamma\beta_m + (1 - \gamma)(\beta_m)^2$, where $\gamma\beta_m$ is the probability that the minor problem is fixed on the consumer's first visit and $(1 - \gamma)(\beta_m)^2$ is the probability that it is fixed on her second visit. Let V denote the present value of the expert's profit when he is active in the market. It follows that

$$V = \frac{\Pi}{n} + \delta[1 - \frac{1}{n}(1 - \alpha)\beta_m(1 - \beta_m)(1 - \gamma)]V. \quad (19)$$

If the expert is active in a period, he receives $\frac{1}{n}$ of the industry profit in that period. The expert will lose all future consumers if he is caught lying in the current period, which occurs with probability $\frac{1}{n}(1 - \alpha)\beta_m(1 - \beta_m)(1 - \gamma)$. So, the expert will survive to the next period with the complementary probability and his expected continuation payoff is the second item in (19). Solving (19), we obtain

$$V = \frac{\Pi}{n - \delta[n - (1 - \alpha)\beta_m(1 - \beta_m)(1 - \gamma)]} = \frac{\alpha(p - c) + (1 - \alpha)(p - c + \varepsilon)(\gamma\beta_m + (1 - \gamma)(\beta_m)^2)}{n - \delta[n - (1 - \alpha)\beta_m(1 - \beta_m)(1 - \gamma)]}. \quad (20)$$

Substituting V and (18) into (6), it follows that

$$\begin{aligned} & \delta(1 - \gamma)(1 - \beta_m) \frac{\alpha(p - c) + (1 - \alpha)(p - c + \varepsilon)(\gamma\beta_m + (1 - \gamma)(\beta_m)^2)}{n - \delta[n - (1 - \alpha)\beta_m(1 - \beta_m)(1 - \gamma)]} \\ &= (p - c + \varepsilon) [\beta_m(1 - \gamma) + \gamma]. \end{aligned} \quad (21)$$

The condition (21) pins down a unique solution $\gamma = 1 - \frac{n(1 - \delta)(p - c + \varepsilon)}{n(1 - \delta)(p - c + \varepsilon)(1 - \beta_m) + \delta\alpha(1 - \beta_m)(p - c)}$. So, given that

all other experts choose the strategy $(p, \beta_m^*(p; k))$ and that the consumer chooses $\gamma = \gamma^*(p; k, n)$, $\beta_m^*(p; k)$ is the expert's best response. Note that for $\gamma^*(p; k, n) > 0$, it is necessary to have $\delta > \underline{\delta}(p; k, n)$. Q.E.D.

Proof for Lemma 3: We first show $\frac{\partial \beta_m^*(p; k)}{\partial k} > 0$. Using (16), we derive

$$\frac{\partial \beta_m^*(p; k)}{\partial k} = \frac{\alpha + (1 - \alpha)\beta_m^*(p; k)}{(1 - \alpha)(p - l_m)(1 - 2\beta_m^*(p; k)) - (1 - \alpha)k}. \quad (22)$$

So, the sign of $\frac{\partial \beta_m^*(p; k)}{\partial k}$ is determined by the sign of the denominator. Substituting (13) into the denominator of (22), we obtain

$$(1 - \alpha)[(p - l_m)(1 - 2\beta_m^*(p; k)) - k] = \sqrt{(1 - \alpha)^2(p - l_m - k)^2 - 4k\alpha(1 - \alpha)(p - l_m)} > 0.$$

Hence, $\frac{\partial \beta_m^*(p; k)}{\partial k} > 0$. Since $\beta_m^*(p; k)$ is independent of n and δ , $\frac{\partial \beta_m^*(p; k)}{\partial n} = \frac{\partial \beta_m^*(p; k)}{\partial \delta} = 0$.

Next, using (15), we derive

$$\frac{\partial \gamma^*(p; k, \delta, n)}{\partial k} = \frac{n(1 - \delta)(p - c + \varepsilon)}{(n(1 - \delta)(p - c + \varepsilon) + \delta\alpha(p - c))(1 - \beta_m^*(p; k))^2} \frac{\partial \beta_m^*(p; k)}{\partial k} > 0, \quad (23)$$

$$\frac{\partial \gamma^*(p; k, \delta, n)}{\partial n} = -\frac{\alpha\delta(1 - \delta)(1 - \beta_m^*(p; k))(p - c)(p - c + \varepsilon)}{[n(1 - \delta)(p - c + \varepsilon)(1 - \beta_m^*(p; k)) + \delta\alpha(1 - \beta_m^*(p; k))(p - c)]^2} < 0, \quad (24)$$

$$\frac{\partial \gamma^*(p; k, \delta, n)}{\partial \delta} = \frac{\alpha n(1 - \beta_m^*(p; k))(p - c)(p - c + \varepsilon)}{[n(1 - \delta)(p - c + \varepsilon)(1 - \beta_m^*(p; k)) + \delta\alpha(1 - \beta_m^*(p; k))(p - c)]^2} > 0. \quad (25)$$

Following (14), we obtain

$$\begin{aligned} \frac{\partial \underline{\delta}(p; k, n)}{\partial k} &= \frac{n\alpha(p - c)(p - c + \varepsilon)}{[n(p - c + \varepsilon)\beta_m^*(p; k) + \alpha(1 - \beta_m^*(p; k))(p - c)]^2} \frac{\partial \beta_m^*(p; k)}{\partial k} > 0. \\ \frac{\partial \underline{\delta}(p; k, n)}{\partial n} &= \frac{\alpha\beta_m^*(p; k)(1 - \beta_m^*(p; k))(p - c)(p - c + \varepsilon)}{[n(p - c + \varepsilon)\beta_m^*(p; k) + \alpha(1 - \beta_m^*(p; k))(p - c)]^2} > 0. \end{aligned}$$

Last, we investigate the impact of k, n and δ on experts' profits. Substituting (15) into (20) and simplifying the expression, we obtain $V = \frac{\alpha(p - c)}{n(1 - \delta)}$. Hence, $\frac{\partial V}{\partial k} = 0$, $\frac{\partial V}{\partial n} < 0$ and $\frac{\partial V}{\partial \delta} > 0$. Q.E.D.

Proof for Proposition 7: Consider the search equilibrium indexed by the price $p^* = c + k$. Substituting $p^* = c + k$ into (13) and (15), it directly follows that $\beta_m^*(c + k; k) \rightarrow 0$ and $\gamma^*(c + k; k, n) \rightarrow 0$ as $k \rightarrow 0$. At $p^* = c + k$, the cutoff discount factor is

$$\underline{\delta}(c + k; k, n) \equiv \frac{n(k + \varepsilon)\beta_m^*(c + k; k)}{n(k + \varepsilon)\beta_m^*(c + k; k) + \alpha(1 - \beta_m^*(c + k; k))k}.$$

Since both the numerator and the denominator of $\underline{\delta}(c + k; k, n)$ converges to 0 as $k \rightarrow 0$, we apply L'Hospital's

Rule

$$\lim_{k \rightarrow 0} \delta(c+k; k, n) = \lim_{k \rightarrow 0} \frac{n\beta_m^*(c+k; k) + n(k+\varepsilon)\partial\beta_m^*(c+k; k)/\partial k}{n\beta_m^*(c+k; k) + n(k+\varepsilon)\partial\beta_m^*(c+k; k)/\partial k + \alpha(1-\beta_m^*(c+k; k)) - \alpha k \partial\beta_m^*(c+k; k)/\partial k} \quad (26)$$

$$= \frac{n\varepsilon \lim_{k \rightarrow 0} \partial\beta_m^*(c+k; k)/\partial k}{n\varepsilon \lim_{k \rightarrow 0} \partial\beta_m^*(c+k; k)/\partial k + \alpha}, \quad (27)$$

where the first equality follows from L'Hospital's Rule and the second equality follows from $\beta_m^*(c+k; k) \rightarrow 0$ as $k \rightarrow 0$.

Next, we derive $\lim_{k \rightarrow 0} \partial\beta_m^*(c+k; k)/\partial k$. Let $N(k)$ denote the numerator of $\beta_m^*(c+k; k)$ and $D(k)$ denote the denominator of $\beta_m^*(c+k; k)$. So,

$$\partial\beta_m^*(c+k; k)/\partial k = \frac{N'(k)D(k) - N(k)D'(k)}{(D(k))^2}. \quad (28)$$

The derivatives $N'(k)$ and $D'(k)$ are

$$\begin{aligned} N'(k) &= \frac{2\alpha(1-\alpha)(c+2k-l_m)}{\sqrt{(1-\alpha)^2(c-l_m)^2 - 4k\alpha(1-\alpha)(c+k-l_m)}} \\ D'(k) &= 2(1-\alpha). \end{aligned}$$

Substitute $N'(k)$ and $D'(k)$ into (28) and take the limit,

$$\lim_{k \rightarrow 0} \partial\beta_m^*(c+k; k)/\partial k = \lim_{k \rightarrow 0} \frac{N'(k)D(k) - N(k)D'(k)}{(D(k))^2} = \frac{\alpha}{(1-\alpha)(c-l_m)}, \quad (29)$$

which follows because $\lim_{k \rightarrow 0} D(k) = 2(1-\alpha)(c-l_m)$, $\lim_{k \rightarrow 0} D'(k) = 2(1-\alpha)$, $\lim_{k \rightarrow 0} N(k) = 0$, and $\lim_{k \rightarrow 0} N'(k) = 2\alpha$.

Substitute (29) into (27), it follows that $\lim_{k \rightarrow 0} \delta(c+k; k, n) = \frac{n\alpha\varepsilon}{n\alpha\varepsilon + \alpha(1-\alpha)(c-l_m)}$. Q.E.D.

Appendix B

Proof for Proposition 3: We use the method proposed by APS (1990) and elaborated in Mailath and Samuelson (2006) to characterize the maximum self-generation set. Since we allow public randomization at the end of each period of the game, the set of the expert's payoff in any Perfect Public equilibria is an interval $[0, \bar{v}]$, where 0 is the expert's payoff in the one-shot game and $\bar{v}$ is the highest possible obtainable profit in a Perfect Public equilibrium. We characterize the largest self-generating set in the following two lemmas. Lemma 5 considers the price range $p \geq c$ and Lemma 6 considers the price range $p < c$. We first denote the five public outcomes $y \in Y \equiv \{1, 2, 3, 4, 5\}$ described in the paragraph following Proposition 3 for future use:

$$y = \begin{cases} 1 & \text{if } R = \emptyset, l = l_m \\ 2 & \text{if } R = \emptyset, l = l_s \\ 3 & \text{if } R = p, a = 1 \\ 4 & \text{if } R = p, a = 0, l = l_m \\ 5 & \text{if } R = p, a_t = 0, l = l_s \end{cases}.$$

Lemma 5 When $p \geq c$, $\bar{v} = \pi^m$ for $\delta \geq \underline{\delta}^m(\alpha)$ and $\bar{v} = 0$ For $\delta < \underline{\delta}^m(\alpha)$.

Since clients are myopic, we focus on the expert's value set. Note that when $\beta_s \leq \beta_m$, upon a treatment recommendation at $p \geq c$, the client's expected loss is at most $E(l) < c \leq p$. So a short-lived consumer's best response is $\gamma = 0$. Let $\Omega_1 \equiv \{p, \{\beta | 0 \leq \beta_s \leq \beta_m \leq 1\}, \gamma = 0\}$ and $\Omega_2 \equiv \{p, \{\beta | 0 \leq \beta_m < \beta_s \leq 1\}, \gamma \in [0, 1]\}$. The action profiles involving short-lived clients' best response are thus $\Omega_1 \cup \Omega_2$. The expert's continuation payoff is a mapping $v : Y \rightarrow [0, \bar{v}]$. Let $V^i, i = 1, 2$, denote the set of payoffs decomposed by Ω_i on $[0, \bar{v}]$.

The proof is divided into three steps. Step 1 characterizes V^1 . Step 2 characterizes V^2 . Step 3 characterizes the maximum self-generating set.

Step 1 Given $\gamma = 0$, the expert's expected payoff is

$$v = \alpha \delta [\beta_s v(5) + (1 - \beta_s) v(2)] + (1 - \alpha) \delta [\beta_m v(4) + (1 - \beta_m) v(1)].$$

Since this strategy profile constitute an equilibrium in the stage game, it is trivially enforceable using a constant continuation $v(5) = v(2) = v(4) = v(1) = \hat{v} \in [0, \bar{v}]$. Thus, $V^1 = [0, \delta \bar{v}]$, where the upper bound is achieved by $\hat{v} = \bar{v}$ and the lower bound is achieved by $\hat{v} = 0$.

Step 2 Given the action profile, the expert's expected payoff is

$$\begin{aligned} v \equiv & \alpha \{ \beta_s [\gamma ((1 - \delta)(p - c) + \delta v(3)) + (1 - \gamma) \delta v(5)] + (1 - \beta_s) \delta v(2) \} + \\ & (1 - \alpha) \{ \beta_m [\gamma ((1 - \delta)(p - c + \varepsilon) + \delta v(3)) + (1 - \gamma) \delta v(4)] + (1 - \beta_m) \delta v(1) \}. \end{aligned} \quad (30)$$

For the expert's recommendation strategy to be optimal, it is necessary that

$$\gamma [(1 - \delta)(p - c + \varepsilon) + \delta v(3)] + (1 - \gamma) \delta v(4) \leq \delta v(1) \quad (31)$$

$$\gamma [(1 - \delta)(p - c) + \delta v(3)] + (1 - \gamma) \delta v(5) \geq \delta v(2). \quad (32)$$

Condition (31) ensures that the expert weakly prefers no treatment for the minor problem and condition (32) says that the expert weakly prefers treatment for the serious problem.

We first characterize the upper bound of V^2 . Using (31) and (32), it is without loss of generality to reduce (30) to

$$v = \alpha \{ \gamma [(1 - \delta)(p - c) + \delta v(3)] + (1 - \gamma)\delta v(5) \} + (1 - \alpha)\delta v(1). \quad (33)$$

The maximum value of V^2 is achieved by solving:

$$\text{Max}_{p, \beta, \gamma, v(\cdot)} \quad (33)$$

Subject to (31), (32),

$$c \leq p \quad (34)$$

$$0 \leq \beta_m < \beta_s \leq 1 \quad (35)$$

$$\gamma = \begin{cases} 1 & \text{for } p < E(l|\beta) \\ [0, 1] & p = E(l|\beta) \\ 0 & p > E(l|\beta) \end{cases}. \quad (36)$$

Condition (36) ensures that γ is the client's best response.

We first show that it is optimal to choose $v(2) = v(4) = 0$ and $v(1) = v(5) = \bar{v}$. The former holds because (33) does not involve $v(2)$ or $v(4)$ and constraints (31) and (32) are relaxed at $v(2) = v(4) = 0$. Next, we show $v(1) = v(5) = \bar{v}$. Note that given $p \geq c$, after substituting $v(2) = 0$, constraint (32) is satisfied for all $v(3), v(5) \in [0, \bar{v}]$ and can be ignored. An increase in $v(1)$ and $v(5)$ weakly relaxes constraint (31) but increases (33). So, it is optimal to choose $v(1) = v(5) = \bar{v}$.

Next, we show $\text{Max } V^2 = \delta \bar{v} \left[1 + \frac{\alpha(1 - \delta)(l_s - c)}{\delta \bar{v} + (1 - \delta)(l_s - c + \varepsilon)} \right]$. Substitute $v(1) = v(5) = \bar{v}$ and $v(2) = v(4) = 0$ into (33) as well as (31) and ignore constraint (32), the expert's expected payoff maximization problem boils down to

$$\text{Max}_{p, \gamma, \beta, v(3)} \alpha \gamma [(1 - \delta)(p - c) + \delta(v(3) - \bar{v})] + \delta \bar{v} \quad (37)$$

subject to (34), (35), (36) and

$$\delta \bar{v} \geq \gamma [(1 - \delta)(p - c + \varepsilon) + \delta v(3)]. \quad (38)$$

Following (38), $v(3) \leq \frac{\bar{v}}{\gamma} - \frac{(1 - \delta)(p - c + \varepsilon)}{\delta}$. So, $v(3) \leq \min\{\frac{\bar{v}}{\gamma} - \frac{(1 - \delta)(p - c + \varepsilon)}{\delta}, \bar{v}\}$. Let $\hat{\gamma} = \frac{\delta \bar{v}}{\delta \bar{v} + (1 - \delta)(p - c + \varepsilon)}$. It can be verified that

$$\min\left\{\frac{\bar{v}}{\gamma} - \frac{(1 - \delta)(p - c + \varepsilon)}{\delta}, \bar{v}\right\} = \begin{cases} \frac{\bar{v}}{\gamma} - \frac{(1 - \delta)(p - c + \varepsilon)}{\delta} & \text{for } \gamma \leq \hat{\gamma} \\ \bar{v} & \text{for } \gamma > \hat{\gamma} \end{cases}. \quad (39)$$

Next, we show the optimal price is $p = E(l|\beta)$. Consider $p = E(l|\beta)$. Since (37) is increasing in γ and $v(3)$, for $\gamma \leq \hat{\gamma}$, (37) is maximized by $v(3) = \bar{v}$ and $\gamma = \hat{\gamma}$, and is

$$\delta \bar{v} \left[1 + \frac{\alpha(1-\delta)(E(l|\beta) - c)}{\delta \bar{v} + (1-\delta)(E(l|\beta) - c + \varepsilon)} \right]. \quad (40)$$

For $\gamma > \hat{\gamma}$, by (39), it is optimal to set $v(3) = \frac{\bar{v}}{\gamma} - \frac{(1-\delta)(p - c + \varepsilon)}{\delta}$. Substitute $v(3)$ into (37), the expert's expected payoff is reduced to

$$-\gamma [\alpha(1-\delta)\varepsilon + \alpha\delta\bar{v}] + \delta\bar{v}(1+\alpha), \quad (41)$$

which is decreasing in γ . Hence, the maximum value of v is achieved by $\gamma = \hat{\gamma}$, which yields the expert (40).

Next, consider $p > E(l|\beta)$. The client's best response is $\gamma = 0$ which yields the expert $\delta\bar{v} < (40)$. Lastly, consider $p < E(l|\beta)$, the client's best response is $\gamma = 1$. By (39), it is optimal to set $v(3) = \bar{v} - \frac{(1-\delta)(p - c + \varepsilon)}{\delta}$. Substitute the optimal values of γ and $v(3)$ into (37), the expert's payoff is $\delta\bar{v} - \alpha(1-\delta)\varepsilon < (40)$.

Lastly, we show it is optimal to set $\beta_m = 0$ and $\beta_s = 1$. Note that (40) is strictly increasing in $E(l|\beta)$ which increases in β_s and decreases in β_m . Thus, it is optimal to have $\beta_s = 1$ and $\beta_m = 0$ which yields $E(l|\beta) = l_s$. As a result,

$$\text{Max } V^2 = \delta\bar{v} \left(1 + \frac{\alpha(1-\delta)(l_s - c)}{\delta\bar{v} + (1-\delta)(l_s - c + \varepsilon)} \right).$$

ii) We characterize the lower bound of V^2 and show $\text{Min } V^2 = 0$. Consider $v(2) = v(3) = v(4) = v(5) = 0$, $p > E(l|\beta)$ and $\gamma = 0$. The expert's payoff is 0 at these values, constraints (31) and (32) are both satisfied, and $\gamma = 0$ is the client's best response. In summary, $V^2 = \left[0, \delta\bar{v} \left(1 + \frac{\alpha(1-\delta)(l_s - c)}{\delta\bar{v} + (1-\delta)(l_s - c + \varepsilon)} \right) \right]$.

Step 3 For $[0, \bar{v}]$ to be a self-generating set, it requires that

$$[0, \bar{v}] \subset \cup_{i=1,2} V^i = \left[0, \delta\bar{v} \left(1 + \frac{\alpha(1-\delta)(l_s - c)}{\delta\bar{v} + (1-\delta)(l_s - c + \varepsilon)} \right) \right].$$

The upper bound $\bar{v}$ is solved from

$$\begin{aligned} \bar{v} &= \delta\bar{v} \left(1 + \frac{\alpha(1-\delta)(l_s - c)}{\delta\bar{v} + (1-\delta)(l_s - c + \varepsilon)} \right) \Rightarrow \\ \bar{v} &= \pi^m = \alpha(l_s - c) - \frac{1-\delta}{\delta}(l_s - c + \varepsilon). \end{aligned}$$

The set $[0, \pi^m]$ is non empty if and only if $\delta \geq \underline{\delta}^m(\alpha)$. Hence, for $p \geq c$, the maximum equilibrium payoff accrued to the expert is π^m for $\delta \geq \underline{\delta}^m(\alpha)$ and is 0 otherwise. Q.E.D.

Lemma 6 When $p \in [c - \varepsilon, c)$, $\bar{v} = \pi^o$ for $\alpha \geq \tilde{\alpha}$ and $\delta \geq \underline{\delta}^o(\alpha)$; $\bar{v} = 0$, otherwise.

We divide action profiles into three categories: i) $\Omega_3 \equiv \{p, \{\beta|0 < \min\{\beta_m, \beta_s\} \leq 1\}, \gamma\}$, ii) $\Omega_4 \equiv \{p, \{\beta|0 = \beta_m \leq \beta_s \leq 1\}, \gamma\}$ and iii) $\Omega_5 \equiv \{p, \{\beta|0 = \beta_s \leq \beta_m \leq 1\}, \gamma\}$. Let $V^i, i = 3, 4, 5$, denote the set of payoffs decomposed by action profile Ω_i on $[0, \bar{v}]$. The proof is divided into 4 steps. The first three steps characterize the sets of payoffs decomposed by action profiles $\Omega_i, i = 3, 4, 5$, respectively. Step 4 characterizes the maximum self-generating set.

Step 1. Consider action profile Ω_3 . For the expert's recommendation strategy to be optimal, it is necessary that

$$\delta v(1) \leq \gamma [(1 - \delta)(p - c + \varepsilon) + \delta v(3)] + (1 - \gamma)\delta v(4), \quad (42)$$

$$\delta v(2) \leq \gamma [(1 - \delta)(p - c) + \delta v(3)] + (1 - \gamma)\delta v(5). \quad (43)$$

Constraints (42) and (43) ensure that the expert prefers treatment for both types of problem. Using (42) and (43), the expert's expected payoff can be written as

$$\begin{aligned} v = & \alpha \{ \gamma [(1 - \delta)(p - c) + \delta v(3)] + (1 - \gamma)\delta v(5) \} \\ & + (1 - \alpha) \{ \gamma [(1 - \delta)(p - c + \varepsilon) + \delta v(3)] + (1 - \gamma)\delta v(4) \}. \end{aligned} \quad (44)$$

We characterize $\max V^3$ by solving the following problem:

$$\max_{p, \beta, \gamma, v(\cdot)} \quad (44)$$

subject to (42), (43)

$$p \in [c - \varepsilon, c) \quad (45)$$

$$0 < \min\{\beta_m, \beta_s\} \leq 1 \quad (46)$$

and (36).

We first show that it is optimal to set $v(1) = v(2) = 0$ and $v(3) = v(4) = v(5) = \bar{v}$. Since $v(1)$ and $v(2)$ are not in (44) and reducing their values relaxes incentive constraints (42) and (43), it is optimal to choose $v(1) = v(2) = 0$. Furthermore, because (44) increases in $v(3)$, $v(4)$ and $v(5)$ and the incentive constraints are

relaxed when these values are increased, it is optimal to set $v(3) = v(4) = v(5) = \bar{v}$. Substitute the optimal value of $v(\cdot)$ into (44) and constraints (42) and (43), the expert's payoff maximization problem is reduced to

$$\text{Max}_{p, \beta, \gamma} \gamma(1 - \delta) \{p - c + (1 - \alpha)\varepsilon\} + \delta\bar{v}, \quad (47)$$

Subject to (45), (46), (36), and

$$0 \leq \gamma(1 - \delta)(p - c + \varepsilon) + \delta\bar{v}, \quad (48)$$

$$0 \leq \gamma(1 - \delta)(p - c) + \delta\bar{v}. \quad (49)$$

, where (48) follows from (42) and (49) follows from (43). We ignore (48) because it is implied by (49). Note that if $\gamma > 0$, then $\beta_s \leq \beta_m = 1$ because (48) holds with strict inequality when (49) is satisfied.

We first consider $\alpha \leq \tilde{\alpha}$ (defined above Proposition 2). We show that $\max V^3 = \delta\bar{v}$ in this case. For (47) to be greater than $\delta\bar{v}$, it is necessary to have $p > c - (1 - \alpha)\varepsilon$ and $\gamma > 0$. But for $\gamma > 0$ to be the consumer's best response, it must follow that $p \leq E(l|\beta_m, \beta_s)$. Because $\beta_m \leq \beta_s = 1$ for $\gamma > 0$, $E(l|\beta_m, \beta_s) \leq E(l)$ and hence $p \leq E(l)$. However, given $\alpha \leq \tilde{\alpha}$, $E(l) \leq c - (1 - \alpha)\varepsilon$, which contradicts $p > c - (1 - \alpha)\varepsilon$. So, $\max V^3 = \delta\bar{v}$, which can be achieved by the action profile $\{p > E(l), \beta_m = \beta_s = 1, \gamma = 0\}$.

Now, we consider $\alpha > \tilde{\alpha}$ and show

$$\max V^3 = \begin{cases} \frac{\delta\bar{v}(1 - \alpha)\varepsilon}{(c - E(l))} & \text{if } \bar{v} < \frac{(1 - \delta)(c - E(l))}{\delta} \\ (1 - \delta) \{E(l) - c + (1 - \alpha)\varepsilon\} + \delta\bar{v} & \text{if } \bar{v} \geq \frac{(1 - \delta)(c - E(l))}{\delta} \end{cases}. \quad (50)$$

First, consider $p = E(l|\beta_m, \beta_s)$. The consumer's best response is $\gamma \in [0, 1]$. Since (47) increases in p and (49) is more relaxed as p increases, it is optimal to set $p = E(l)$ which is obtained when $\beta_m = \beta_s = 1$.

Constraint (49) is satisfied if $\gamma \leq \bar{\gamma} \equiv \min\{1, \frac{\delta\bar{v}}{(1 - \delta)(c - E(l))}\}$, where

$$\bar{\gamma} = \begin{cases} \frac{\delta\bar{v}}{(1 - \delta)(c - E(l))} & \text{if } \bar{v} < \frac{(1 - \delta)(c - E(l))}{\delta} \\ 1 & \text{if } \bar{v} \geq \frac{(1 - \delta)(c - E(l))}{\delta} \end{cases}. \quad (51)$$

Because (47) increases in γ , the expert's payoff is maximized at $\bar{\gamma}$ and is (50).

Next, consider $p < E(l|\beta_m, \beta_s)$. The consumer's best response is $\gamma = 1$. The constraint (49) is satisfied when $\bar{v} \geq \frac{(1 - \delta)(c - E(l))}{\delta}$ and the expert's payoff is bounded above by $(1 - \delta) \{E(l) - c + (1 - \alpha)\varepsilon\} + \delta\bar{v}$ by the same argument as in the case of $p = E(l|\beta_m, \beta_s)$.

First, consider $p > E(l|\beta_m, \beta_s)$. The consumer's best response is $\gamma = 0$, yielding the expert a payoff $\delta\bar{v}$.

Given $\alpha > \tilde{\alpha}$, $\delta\bar{v}$ is smaller than (50) for all $\bar{v}$.

Next, we derive the lower bound of V^3 . It can be verified that $\min V^3 = 0$, which is obtained by $v(1) = v(2) = v(3) = v(4) = v(5) = 0$, $\gamma = 0$ and $p > E(l|\beta)$. In summary,

$$V^3 = \begin{cases} \left[0, \frac{\delta\bar{v}(1-\alpha)\varepsilon}{(c-E(l))} \right] & \text{if } \bar{v} < \frac{(1-\delta)(c-E(l))}{\delta} \\ 0, (1-\delta)\{E(l)-c+(1-\alpha)\varepsilon\} + \delta\bar{v} & \text{if } \bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta} \end{cases}. \quad (52)$$

Step 2 Consider action profile Ω_4 . Given $p < c$ and $\beta_m = 0$, the consumer's best response is $\gamma = 1$.

Substitute $\gamma = 1$ and $\beta_m = 0$ into (30), the expert's payoff is reduced to

$$\alpha\{\beta_s[(1-\delta)(p-c) + \delta v(3)] + (1-\beta_s)\delta v(2)\} + (1-\alpha)\delta v(1). \quad (53)$$

We first show $\text{Max } V^4 = \delta\bar{v}$. The maximum value of V^4 is obtained by the following maximization program:

$$\max_{v(1), v(2), v(3), p, \beta_s} \quad (53)$$

subject to (45), (36)

$$0 \leq \beta_s \leq 1 \quad (54)$$

$$\delta v(1) \geq [(1-\delta)(p-c+\varepsilon) + \delta v(3)], \quad (55)$$

where (55) ensures $\beta_m = 0$.

We show it is optimal to have $v(1) = v(2) = \bar{v}$. To see this, an increase in $v(1)$ or $v(2)$ weakly relaxes (55) and strictly increases (53), so $v(1) = v(2) = \bar{v}$. Substitute $v(1) = v(2) = \bar{v}$ into (53) and rearrange it, (53) can be reduced to

$$\delta\bar{v} - \alpha\beta_s[(1-\delta)(c-p) + \delta(\bar{v} - v(3))]. \quad (56)$$

Because $p < c$ and $v(3) \leq \bar{v}$, (56) is maximized by $\beta_s = 0$. Note that (55) can be satisfied by $v(3) = 0$ and $p = c - \varepsilon$. So, $\text{Max } V^4 = \delta\bar{v}$.

Next, we show $\min V^4 = 0$. The minimum value of V^4 is solved by minimizing (53) subject to (45), (36), (54) and (55). It can be verified that when $\beta_s = 0$, $v(2) = v(3) = 0$, $p = c - \varepsilon$ and $v(1) = \frac{(1-\delta)(p-c+\varepsilon)}{\delta} = 0$, the expert's payoff is 0. In summary $V^4 = [0, \delta\bar{v}]$.

Step 3 Consider action profile Ω_5 . Given $0 = \beta_s \leq \beta_m$ and $l_m < c - \varepsilon \leq p$, the client's best response is $\gamma = 0$. For the expert's recommendation strategy to be optimal, it is necessary that

$$\gamma[(1 - \delta)(p - c) + \delta v(3)] + (1 - \gamma)\delta v(5) \leq \delta v(2). \quad (57)$$

Substitute $\beta_s = 0$, $\gamma = 0$ into (30), it is reduced to

$$\alpha \delta v(2) + (1 - \alpha) \delta \{ \beta_m v(4) + (1 - \beta_m) v(1) \}. \quad (58)$$

We choose β_m , p and $v(\cdot)$ to maximize (58) subject to $\beta_m \in [0, 1]$, (45), (36), and

$$v(5) \leq v(2), \quad (59)$$

where (59) follows from (57) after substituting $\beta_s = 0$ and $\gamma = 0$. It is optimal to set $v(2) = v(4) = v(1) = \bar{v}$ which yields $\max V^5 = \delta \bar{v}$. It can be verified that $\min V^5 = 0$ which is obtained by $v(1) = v(2) = v(3) = v(4) = v(5) = 0$, $p > E(l|p)$ and $\gamma = 0$. Hence, $V^5 = [0, \delta \bar{v}]$

Step 4 We characterize the maximum self-generating set. The set $[0, \bar{v}]$ is self-generating if and only if $[0, \bar{v}] \subset \cup_{i=3,4,5} V^i$. When $\alpha \leq \tilde{\alpha}$, $\cup_{i=3,4,5} V^i = [0, \delta \bar{v}]$. The condition $[0, \bar{v}] \subset [0, \delta \bar{v}]$ is satisfied only at $\bar{v} = 0$.

When $\alpha > \tilde{\alpha}$, the union $\cup_{i=3,4,5} V^i$ is (52). Suppose $\bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}$. The upper bound $\bar{v}$ is solved from

$$\begin{aligned} \bar{v} &= (1 - \delta) \{ E(l) - c + (1 - \alpha) \varepsilon \} + \delta \bar{v} \Leftrightarrow \\ \bar{v} &= E(l) - c + (1 - \alpha) \varepsilon. \end{aligned}$$

The maximum value is sustainable if and only if

$$\begin{aligned} E(l) - c + (1 - \alpha) \varepsilon &\geq \frac{(1 - \delta)(c - E(l))}{\delta} \\ \delta &\geq \underline{\delta}^o(\alpha). \end{aligned}$$

Suppose $\bar{v} < \frac{(1 - \delta)(c - E(l))}{\delta}$, the upper bound $\bar{v}$ is solved from $\bar{v} = \frac{\delta \bar{v}(1 - \alpha) \varepsilon}{(c - E(l))}$, which gives a unique solution $\bar{v} = 0$. We conclude that $\bar{v} = \pi^o$ for $\alpha > \tilde{\alpha}$ and $\delta \geq \underline{\delta}^o(\alpha)$ and 0 otherwise. Q.E.D.

Proof for Proposition 5: In the most profitable equilibrium, consumer surplus is zero. If there exists an equilibrium more efficient than the most profitable equilibrium, consumer surplus in the equilibrium must

be positive. Let $\underline{u} \in (0, \alpha(l_s - c)]$ denote the minimum utility consumers must receive in an equilibrium. The proof is divided into two steps.

Step 1 shows that the monitoring-by-rejection equilibrium is the most efficient equilibrium when $p \geq c$. We begin by characterizing the expert's maximum payoff given $\underline{u}$. For the consumer to receive a positive surplus, $p < E(l|\beta_m, \beta_s)$ must hold, which implies $\gamma = 1$. The proof for Lemma 5 shows that $p < E(l|\beta_m, \beta_s)$ can only be supported by the action profile Ω_2 . We modify the maximization problem in Step 2 of Lemma 5 by adding the constraint on the consumer's payoff

$$(\alpha\beta_s + (1 - \alpha)\beta_m)(E(l|\beta_m, \beta_s) - p) \geq \underline{u}. \quad (60)$$

First, ignore (60) and substitute $\gamma = 1$ into the maximization program. Using the same argument as in the case of $p < E(l|\beta_m, \beta_s)$ in Step 2 of the proof for Lemma 5, the expert's maximum payoff is $\delta\bar{v} - \alpha(1 - \delta)\varepsilon$. Now, we consider (60). Note that because the functions $v(\cdot)$ do not affect (60), they take the same values as in the proof for Lemma 5. Moreover, since the expert's maximum payoff does not involve p or β , we can set $p = c$ and $(\beta_m = 0, \beta_s = 1)$ so that (60) and $p < E(l|\beta_m, \beta_s)$ are both satisfied. Hence, given $\underline{u}$, the expert's maximum payoff is $\delta\bar{v} - \alpha(1 - \delta)\varepsilon$. The self-generating set requires $[0, \bar{v}] \subset [0, \delta\bar{v} - \alpha(1 - \delta)\varepsilon]$, which cannot be satisfied by any $\bar{v}$. This shows that there does not exist an equilibrium in which $p \geq c$ and $\underline{u} > 0$.

Step 2 shows that the one-price-fix-all equilibrium is the most efficient equilibrium for $c - \varepsilon \leq p < c$. We first characterize the expert's maximum payoff given the consumer's reservation utility $\underline{u}$. We consider the action profile in which $p < E(l|\beta_m, \beta_s)$ and hence rule out Ω_5 . In the following analysis, we focus on action profiles Ω_3 and Ω_4 .

First, consider Ω_3 . The expert's problem is the same as in Step 1 of the proof for Lemma 6 except that we add the new constraint (60) and substitute $\gamma = 1$ in the constrained maximization program. So, the expert's maximization problem is reduced to

$$\text{Max}_{p, \beta_m, \beta_s} (1 - \delta) \{p - c + (1 - \alpha)\varepsilon\} + \delta\bar{v}, \quad (61)$$

subject to (60), $0 \leq \beta_s \leq \beta_m = 1$, and

$$0 \leq (1 - \delta)(p - c) + \delta\bar{v}. \quad (62)$$

Because (61) increases in p and (62) is relaxed when p increases, it is optimal to set $p = E(l|\beta_m, \beta_s) - \underline{u}$ which is derived when (60) binds. Because $E(l|\beta_m, \beta_s)$ increases in β_s , it is optimal to have $\beta_s = 1$. Hence, the expert's maximum payoff is

$$(1 - \delta) \{E(l) - c - \underline{u} + (1 - \alpha)\varepsilon\} + \delta\bar{v},$$

provided that $\bar{v} \geq \frac{(1-\delta)(c-E(l)+\underline{u})}{\delta}$ which follows from (62). In summary, given $\underline{u}$,

$$V^3 = [0, (1 - \delta) \{E(l) - c - \underline{u} + (1 - \alpha)\varepsilon\} + \delta\bar{v}].$$

Now, consider Ω_4 . We modify the expert's constrained maximization problem in Step 2 of the proof for Lemma 6 by adding the consumer's participation constraint $\alpha\beta_s(l_s - p) \geq \underline{u}$, which is obtained by substituting $\beta_m = 0$ into (60). Following the same analysis as in Step 2 of the proof for Lemma 6, the expert's expected payoff is (56). Since (56) increases in $v(3)$, it is optimal to set $v(3) = \bar{v} - \frac{(1 - \delta)(p - c + \varepsilon)}{\delta}$ so that (55) binds. Moreover, because (56) decreases in β_s , it is optimal to set $\beta_s = \frac{\underline{u}}{\alpha(l_s - p)}$ which is obtained when the consumer's participation constraint binds. Substituting $v(3)$ and β_s into (56), it is reduced to $\delta\bar{v} - \frac{\underline{u}(1-\delta)\varepsilon}{l_s - p}$ which decreases in p . So, $\max V^4 = \delta\bar{v} - \frac{\underline{u}(1-\delta)\varepsilon}{l_s - c + \varepsilon}$, which is achieved by $p = c - \varepsilon$.

Next, we characterize the maximum payoff accrued to the expert given $\underline{u}$. The union $\cup_{i=3,4} V^i = [0, \delta\bar{v} + (1 - \delta) \{E(l) - c - \underline{u} + (1 - \alpha)\varepsilon\}]$, provided that $\underline{u} \leq E(l) - c + (1 - \alpha)\varepsilon$. The upper bound $\bar{v}$ is solved from

$$\bar{v} = \delta\bar{v} + (1 - \delta) \{E(l) - c - \underline{u} + (1 - \alpha)\varepsilon\} \Rightarrow$$

$$\bar{v} = E(l) - c - \underline{u} + (1 - \alpha)\varepsilon.$$

So, given $\underline{u}$, the maximum total surplus is $\underline{u} + \bar{v} = E(l) - c + (1 - \alpha)\varepsilon$, which is the total surplus of the one-price-fix-all equilibrium. Lastly, if $\underline{u} > E(l) - c + (1 - \alpha)\varepsilon$, the maximum of the union $\cup_{i=3,4} V^i$ is strictly less than $\delta\bar{v}$. The requirement $[0, \bar{v}] \subset \cup_{i=3,4} V^i$ cannot be satisfied for any $\bar{v}$, so there does not exist an equilibrium in which $\underline{u} > E(l) - c + (1 - \alpha)\varepsilon$. We conclude that when $p \in [c - \varepsilon, c)$, the highest total surplus which can be supported by any equilibrium is $E(l) - c + (1 - \alpha)\varepsilon$. Q.E.D.

References

- [1] **Abreu, Dilip, David Pearce, and Ennio Stacchetti.** 1990. "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring." *Econometrica*, **58**: 1041-1063.
- [2] **Alger, Ingela, and Francois. Salanie.** 2006. "A Theory of Fraud and Over-treatment in Experts Markets." *Journal of Economics and Management Strategy*, **15**: 853-881.
- [3] **Bardey, David , Denis Gromb, David Martimort, and Jerome Pouyet.** 2018. "Controlling Sellers Who Provide Advice: Regulation and Competition." working paper
- [4] **Bar-Isaac, Heski, and Steven Tadelis.** 2008. "Seller reputation." *Foundations and Trends in Microeconomics*, **4**: 273-351.
- [5] **Bester, Helmut, and Matthias Dahm.** 2018. "Credence Goods, Costly Diagnosis and Subjective Evaluation." *The Economic Journal*, **128**:1367–1394.
- [6] **Chen, Yongmin, Jianpei Li, and Jin Zhang.** 2018. "Liability in Markets for Credence Goods." working paper.
- [7] **Darby, Michael R., and Edi Karni.** 1973. "Free Competition and the Optimal Amount of Fraud." *Journal of Law and Economics* **16**: 67-88.
- [8] **Dulleck, Uwe, and Rudolf Kerschbamer.** 2006. "On Doctors, Mechanics, and Computer Specialists: the Economics of Credence Goods." *Journal of Economic Literature*, **44**: 5-42.
- [9] **Dulleck, Uwe, and Rudolf Kerschbamer.** 2009. "Experts vs. Discounters: Consumers Free-Riding and Experts Withholding Advice in Markets for Credence Goods." *International Journal of Industrial Organization*, **27**: 15–23.
- [10] **Dulleck, Uwe., Rudolf Kerschbamer, and Matthias Sutter.** 2011. "The Economics of Credence Goods: an Experiment on The Role of Liability, Verifiability, Reputation, and Competition." *American Economic Review*, **101**: 526–555.

- [11] **Ely, Jeffrey, and Juuso Välimäki.** 2003. “Bad Reputation.” *Quarterly Journal of Economics* **118**: 785-814.
- [12] **Emons, Winand.** 1997. “Credence Goods and Fraudulent Experts.” *The RAND Journal of Economics*, **28**: 107-119.
- [13] **Emons, Winand.** 2001. “Credence Goods Monopolists.” *International Journal of Industrial Organization*, **19**: 375-389.
- [14] **Fong, Yuk-fai.** 2005. “When do Experts Cheat and Whom do They Target.” *The RAND Journal of Economics*, **36**: 113-130.
- [15] **Fong, Yuk-fai, and Ting Liu.** 2018. “Liability and Reputation in Credence Goods Markets.” *Economics Letters*, **166**: 35-39.
- [16] **Fong, Yuk-fai., Ting Liu, and Donald Wright.** 2014. “On the Role of Verifiability and Commitment in Credence Goods Markets.” *International Journal of Industrial Organization*, **37**: 118-129.
- [17] **Frankel, Alexander, and Michael Schwarz.** 2014. “Experts and Their Records.” *Economic Inquiry*, **52**: 56-71.
- [18] **Hafner, Samuel, and Curtis R. Taylor.** 2017. “Advice is Cheap: Information is Not!” working paper
- [19] **Hörner, Johannes.** 2002. “Reputation and Competition.” *American Economic Review*, **92**: 644-663.
- [20] **Liu, Ting.** 2011. “Credence Goods Markets with Conscientious and Selfish Experts.” *International Economic Review*, **52**: 227-244.
- [21] **Liu, Ting, Estefania Vergara-Cobos, and Yiyi Zhou.** 2018. “Pricing Schemes and Seller Fraud: Evidence from New York City Taxi Rides.” working paper.
- [22] **Klein, Benjamin, and Keith Leffler.** 1981. “The Role of Market Forces in Assuring Contractual Performance.” *Journal of Political Economy*, **89**: 615-641.

- [23] **Mailath, George, and Larry Samuelson.** 2006. "Repeated Games and Reputations." Oxford University Press.
- [24] **Mimra, Wanda, Alexander Rasch, and Christian Waibel.** 2016. "Price Competition and Reputation in Credence Goods Markets: Experimental Evidence." *Games and Economic Behavior*, **100**: 337-352.
- [25] **Park, In-Uck.** 2005. "Cheap Talk Referrals of Differentiated Experts in Repeated Relationship." *The RAND Journal of Economics*, **36**: 391-411.
- [26] **Pesendorfer, Wolfgang, and Asher Wolinsky.** 2003. "Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice." *The Review of Economic Studies*, **70**: 417-437.
- [27] **Pitchik, Carolyn, and Andrew Schotter.** 1987. "Honesty in a Model of Strategic Information Transmission." *American Economic Review*, **77**: 1032-36.
- [28] **Sihvonen, Raine, et. al.** 2013. "Arthroscopic Partial Meniscectomy versus Sham Surgery for a Degenerative Meniscal Tear." **369**: 2515-2524.
- [29] **Schneider, Henry.** 2012. "Agency Problems and Reputation in Expert Services: Evidence from Auto Repair." *Journal of Industrial Economics*, **60**: 406-433.
- [30] **Taylor, Curtis R.** 1995. "The Economics of Breakdowns, Checkups, and Cures." *Journal of Political Economy*, **103**: 53-74.
- [31] **Wolinsky, Asher.** 1993. "Competition in a Market for Informed Experts' Services." *The RAND Journal of Economics*, **24**: 380-398.
- [32] **Wolinsky, Asher.** 1995. "Competition in markets for credence goods." *Journal of Institutional and Theoretical Economics*, **151**: 117-131.