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Transfer Function Analysis of the Power Supply Rejection Ratio of Capacitor-Less LDOs

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Abstract—This paper presents a rigorous transfer function analysis of the power supply rejection ratio (PSRR) of typical capacitor-less low-dropout (LDO) regulators. An efficient and accurate model is proposed to relieve the complication. The transfer functions, together with the poles and zeros, are obtained regarding a typical capacitor-less LDO topology as an example. The numerical results are plotted and compared to circuit simulation results using a commercial 65-nm technology for verification, and using the proposed model the mechanisms of different PSRR enhancement techniques are discussed. The proposed model and derived transfer functions match well with the simulation and measurement results in prior works.

Keywords—Low-dropout regulator, power supply rejection ratio, transfer function, poles and zeros

I. INTRODUCTION

The extensive demand for mobile devices, such as cell-phones and wireless handsets, is calling for efficient on-chip power management units (PMUs). Conventional power management units, as Fig. 1 shows, consist of efficient switching converters followed by low drop-out (LDO) regulators for further finer regulation to power up noise-sensitive analog and RF blocks. To reduce the PCB area for SoC applications, the switching converter is pushing its switching frequency to MHz range for smaller off-chip inductor, and the LDO also turns to capacitor-less, eliminating the need for off-chip output capacitors to facilitate integration.

However, this poses a potential threat to noise-sensitive analog and RF blocks. The switching converter inevitably introduces ripples at switching frequency and its harmonics as indicated in Fig. 1, making the ripple even harder to suppress. Furthermore, the capacitor-less LDOs are also intrinsically inferior to their with-capacitor counterparts in terms of the power supply rejection ratio (PSRR) performance.

Various PSRR enhancement techniques have been reported during the last decade [1]–[7]. However, the fundamental facts about the PSRR of generic capacitor-less LDOs have not been studied in detail. An intuitive analysis has been given in [8] but it is far from the desired detailed transfer function analysis. Recent works [3], [5] have presented some simple analysis on the PSRR, but only to sub-MHz frequency range because the parasitic capacitors of the power transistor are not included. An attempt has been demonstrated in [9], but the results are not verified and the analysis is not based on capacitor-less LDOs, making it less useful for the state-of-the-art designs. This paper proposes an efficient and accurate model of the PSRR of capacitor-less LDOs and obtains the transfer function

Fig. 1. Block diagram of a typical power management unit and the requirement for high-PSRR LDOs especially over MHz frequency range.

of the PSRR with poles and zeros for insightful discussions on various PSRR enhancement techniques and conclusions.

II. NOTATION DECLARATIONS AND ASSUMPTIONS

A. Notation Declarations

As illustrated in Fig. 2, g_{mi} , r_{oi} and C_{pi} denote the transconductance, output resistance and lumped output parasitic capacitance of the i th stage, respectively. C_c is the frequency compensation capacitor.

g_{mp} , r_{ds} and g_{ds} represent the transconductance, output resistance and output transconductance of the power transistor. C_{gs} , C_{gd} and C_{ds} are the corresponding parasitic capacitors. Since *body* is always connected to *source*, C_{gb} is eliminated by integrating it to C_{gs} and likewise, C_{db} to C_{ds} .

B. Assumptions

Some reasonable assumptions are made to simplify the derivation without compromising accuracy.

- 1) The finite PSRR of the bandgap reference and the error amplifier are neglected.
- 2) The voltage gain of each stage, excluding the power stage, is much greater than unity ($g_{mi}r_{oi} \gg 1$).
- 3) The output load capacitor, the compensation capacitor and the parasitic capacitors of the power transistor are much larger than the lumped output parasitic capacitors of each stage (C_L , C_c , C_{gs} , C_{gd} and $C_{ds} \gg C_{pi}$).
- 4) The feedback resistor string has a large resistance and can be neglected ($R_F = R_{F1} + R_{F2} \gg R_L$ and r_{ds}).

III. THE PROPOSED MODEL OF THE PSRR OF LDO

It is aggressive yet unrewarding to just work out the transfer functions based on the complete small-signal circuit. Even if it is managed to do so, the results would be so complicated that no constructive conclusions beneficial to IC designers can be

Fig. 2. Notation declarations and the simulation setup for the open-loop PSRR, excluding the frequency compensation network.

Fig. 3. The small-signal circuit for the derivation of the transfer function of the open-loop PSRR, excluding the frequency compensation network.

drawn. This is due to i) the complexity of the regulation loop, and ii) the inclusion of the parasitic capacitors of the power transistor like C_{gs} , C_{gd} and C_{ds} , without which the transfer functions are confined to a narrow range of frequency.

The complexity of the regulation loop is eliminated by turning to the open-loop PSRR, and the parasitic capacitors are processed with the method introduced later with no overhead.

A. The Open-Loop PSRR

Because of the existence of the regulation loop, the PSRR that is usually considered and measured is in fact the closed-loop PSRR, which is overwhelmingly complex. However, the closed-loop transfer function can always be acquired by deriving the open-loop transfer function, $PSRR_{OL}(s)$, and the loop gain function, $T(s)$, respectively. According to equation (1), the closed-loop transfer function can be obtained. This “divide and conquer” method makes it possible to work out the transfer function of the closed-loop PSRR.

$$PSRR_{CL}(s) = \frac{PSRR_{OL}(s)}{1 + T(s)} \quad (1)$$

Most published PSRR enhancement techniques focus either on boosting up the loop gain [1] or the loop bandwidth [2] of the regulation loop, or improving the open-loop PSRR using feed-forward ripple cancellation methods [3]–[5], [7]. Thus, it is reasonable to decompose the closed-loop PSRR into the open-loop PSRR and the loop gain function, so that not only the complexity is relieved but it is also easier to evaluate the performance of various PSRR enhancement techniques.

Fig. 4. (a) The original circuit with a coupling capacitor C_f ; (b) The equivalent transformation applied to the coupling capacitor; (c) The equivalent circuit after the transformation is applied to the coupling capacitor C_f .

Investigating the open-loop PSRR requires the regulation loop to be broken only in AC domain, which is hardly feasible in the real world. However, in software simulations, it is possible as illustrated in Fig. 2. The impossibly large and ideal inductor cut off AC signals while leave DC connected, satisfying the need for the simulation of the open-loop PSRR.

B. The AC Coupling Capacitors

Apart from the complexity of the regulation loop, parasitic capacitors like C_{gs} , C_{gd} and C_{ds} acting as AC coupling capacitors in Fig. 3 bring further complication. An efficient pre-processing, if any, would be favorable to the derivation of the open-loop PSRR.

A typical coupling capacitor in a common circuit is shown in Fig. 4(a). The coupling capacitor couples node X with node Y, complicating the calculation of the node voltages and the transfer functions using KCL equations. Nevertheless, by applying the equivalent transformation introduced in Fig. 4(b) inspired by [5], the circuit can be simplified to Fig. 4(c).

As displayed, the coupling capacitor is decomposed into four components, a loading capacitor and a voltage-controlled current source regarding each node. There is no deviation if the node voltages are calculated based on Fig. 4(a) and Fig. 4(c) respectively and compare. The most obvious changes are i) the total number of components is reduced, and ii) there is no component or direct electrical connection between node X and node Y. These changes greatly facilitate the derivation of the node voltages and the transfer functions, especially in the case of Fig. 3 where there are three coupling capacitors. Furthermore, it turns out that coupling resistors like r_{ds} can also be transformed in exactly the same way, thus this transformation can help eliminate four coupling components in Fig. 3, that is, C_{gs} , C_{gd} , C_{ds} and r_{ds} .

IV. THE TRANSFER FUNCTION OF THE PSRR OF LDO

The derivation of the transfer functions are carried out on a typical capacitor-less LDO with a two-stage error amplifier and simple Miller compensation in 65-nm CMOS process as shown in Fig. 5. The compensation capacitor C_c occupies the same place as C_{gd} , thus $C_{gd} + C_c$ is denoted as C_{gdt} .

$$\text{PSRR}_{\text{OL}}(s) \approx \frac{(g_{mp} + g_{ds})R_{LT} \left[1 + sr_{o2} \left(C_g - \frac{g_{mp}}{g_{mp} + g_{ds}} C_{gs} \right) + s^2 \frac{r_{o2}}{g_{mp} + g_{ds}} (C_{ds} C_g + C_{gdt} C_{gs}) \right]}{1 + sr_{o2} (C_g + g_{mp} R_{LT} C_{gdt}) + s^2 r_{o2} R_{LT} (C_g C_{LT} - C_{gdt}^2)} \quad (2)$$

$$p_1 \approx -\frac{1}{r_{o2} (C_g + g_{mp} R_{LT} C_{gdt})}, \quad p_2 \approx -\frac{C_g + g_{mp} R_{LT} C_{gdt}}{R_{LT} (C_g C_{LT} - C_{gdt}^2)}, \quad z_1 \approx -\frac{1}{r_{o2} \left(C_g - \frac{g_{mp}}{g_{mp} + g_{ds}} C_{gs} \right)}, \quad z_2 \approx -\frac{g_{ds} C_g + g_{mp} C_{gdt}}{C_{ds} C_g + C_{gdt} C_{gs}}$$

Fig. 5. The topology of a typical capacitor-less LDO with Miller compensation for analysis and simulation verification.

A. The Open-Loop PSRR

The small signal circuit for the open-loop PSRR is shown in Fig. 6. The complexity is greatly reduced by eliminating the regulation loop and the coupling components, and based on the KCL equations the transfer function can be acquired. The transfer function and the detailed poles and zeros are indicated in equation (2) shown on the top of this page.

B. The Loop Gain Function of the Regulation Loop

Referring to Fig. 3, conventionally the loop gain of the Miller-compensated regulation loop can be expressed as

$$T(s) = \frac{v_{t,out}(s)}{v_{t,in}(s)} = \frac{A_{DC}}{(1 + \frac{s}{p_d})(1 + \frac{s}{p_{nd}})}, \quad \text{where} \quad (3)$$

$$A_{DC} = \beta g_{m1} r_{o1} g_{m2} r_{o2} g_{mp} (R_L \parallel r_{ds})$$

$$p_d \approx \frac{1}{r_{o2} C_c g_{mp} (R_L \parallel r_{ds})}, \quad p_{nd} \approx \frac{g_{mp}}{C_L}$$

The non-dominant pole p_{nd} is placed to be higher than the loop bandwidth, preserving single-pole roll-off characteristics and sufficient phase margin. Based on the loop gain function of the regulation loop, $\frac{1}{1+T(s)}$ could be obtained as part of the transfer function of the closed-loop PSRR.

$$\frac{1}{1+T(s)} \approx \frac{\left(1 + \frac{s}{p_d}\right) \left(1 + \frac{s}{p_{nd}}\right)}{A_{DC} \left(1 + \frac{s}{A_{DC} p_d} + \frac{s^2}{A_{DC} p_d p_{nd}}\right)} \approx \frac{\left(1 + \frac{s}{p_d}\right) \left(1 + \frac{s}{p_{nd}}\right)}{A_{DC} \left(1 + \frac{1}{Q_{tx}} \frac{s}{p_{tx}} + \frac{s^2}{p_{tx}^2}\right)} \quad (4)$$

Note that a pair of complex poles emerges at the frequency of $p_{tx} = \sqrt{A_{DC} p_d p_{nd}}$ with a Q factor of $Q_{tx} = \sqrt{\frac{A_{DC} p_d}{p_{nd}}}$.

Fig. 6. The simplified small signal circuit to derive the transfer function of the open-loop PSRR.

C. The Closed-Loop PSRR

The transfer function of the closed-loop PSRR can be acquired according to equation (1). The final results are given as equation (5), shown on the top of next page.

D. Simulation Verification

The circuit in Fig. 5 is simulated and the numerical circuit parameters are extracted to plot the derived transfer functions, which are compared to the simulation results for verification as demonstrated in Fig. 7, where poles and zeros are explicitly marked. It can be seen that the numerical plots match the simulation results well up to z_2 (>100 MHz), above which the accuracy degrades significantly because other tiny parasitic capacitors start to conduct.

The development from the open-loop PSRR to the closed-loop PSRR is shown in Fig. 8, with a detailed inheritance of poles and zeros demonstrated. At low frequency, the closed-loop PSRR follows the loop gain function. In mid-frequency, a small peak is observed due to the complex poles p_{tx} from $\frac{1}{1+T(s)}$, and at high frequency the closed-loop PSRR follows the open-loop PSRR since the denominator is unity.

E. Discussions on Various PSRR Enhancement Techniques

Most published PSRR enhancement techniques fall into one of the three categories: i) boosting up the loop gain [1], or ii) boosting up the loop bandwidth [2], or iii) improving the open-loop PSRR using feed-forward ripple cancellation (FFRC) [3]–[5], [7]. The detailed mechanisms are demonstrated in Fig. 9 based on the proposed model.

Boosting up the loop gain only helps with the low-frequency PSRR because it is confined around the frequency of z_1 , which is consistent with the existing conclusions. By comparison, boosting up the loop bandwidth gives a wide-frequency PSRR performance, and the two peaks in the measurement results in [2] are successfully predicted by the proposed model. The tri-loop control gives rise to one peak between 1 MHz and 10

$$\begin{aligned}
\text{PSRR}_{\text{CL}}(s) &= \frac{\text{PSRR}_{\text{OL}}(s)}{1 + T(s)} \approx \frac{(g_{mp} + g_{ds})R_{LT} \left(1 + \frac{s}{z_1}\right) \left(1 + \frac{s}{z_2}\right)}{\left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right)} \frac{\left(1 + \frac{s}{p_d}\right) \left(1 + \frac{s}{p_{nd}}\right)}{A_{\text{DC}} \left(1 + \frac{1}{Q_{tx}} \frac{s}{p_{tx}} + \frac{s^2}{p_{tx}^2}\right)} \quad (p_1 \approx p_d) \\
&\approx \frac{g_{mp} + g_{ds}}{g_{mp}} \frac{1}{g_{m1}r_{o1}g_{m2}r_{o2}} \frac{\left(1 + \frac{s}{z_1}\right) \left(1 + \frac{s}{p_{nd}}\right) \left(1 + \frac{s}{z_2}\right)}{\left(1 + \frac{1}{Q_{tx}} \frac{s}{p_{tx}} + \frac{s^2}{p_{tx}^2}\right) \left(1 + \frac{s}{p_2}\right)} \quad (5)
\end{aligned}$$

Fig. 7. The simulation results (black) and the Matlab plots (red) of the open-loop PSRR with difference load current levels.

Fig. 8. The development from the open-loop PSRR (blue) to the closed-loop PSRR (red), with the inheritance of poles and zeros demonstrated.

MHz (GBW of loop 1 & 2) and another peak between 1 GHz and 10 GHz (GBW of loop 3) due to the complex poles.

Prior designs using FFRC [3]–[5], [7] modulate the open-loop PSRR rather than the loop gain function. This method manipulates the zeros of the open-loop PSRR while leaving the poles intact. By pushing z_1 to higher frequency, it is possible to have a good PSRR performance up to mid-frequency (~ 10 MHz), and the “valley” region is also successfully predicted in Fig. 9 which can also be observed in the simulation and measurement results in [3]–[5], [7]. The frequency range where the “valley” emerges strongly depends on the circuit parameters thus the figure is only for intuitive perception.

V. CONCLUSIONS

An efficient and accurate model to analyze the PSRR of typical capacitor-less LDOs is proposed in this paper. The derived transfer functions match well with the simulation results,

Fig. 9. The mechanisms of difference PSRR enhancement techniques illustrated based on the proposed model and the derived transfer functions.

and discussions on various PSRR enhancement techniques are carried out based on the proposed model.

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